GCSE Number Rules and Tools

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About this booklet

This booklet covers all the number work you will need for a foundation GCSE in Maths.

It is not a textbook and it is not a revision workbook, it just explains the rules that you need to know.

I hope this booklet will help you organise your study and I hope it will help you revise.

You will find mistakes, as this is the first time I have used this booklet. Please let me know if you find a mistake or a typing error.

Keith Burnett, B102.

Symbols and conventions

Symbol with example	Meaning	Notes and associated words
3 + 4	Adding, addition	Sum, total
11 – 6	Subtracting, subtraction	Difference
2 × 5	Multiply, multiplication	Product
15 ÷ 5	Divide, division	Quotient (rare!)
15.239261 ≈ 15	Approximately, approximation	Estimate, round off, round up
5 > 3	Greater than	
2 < 1000	Less than	
21 ≥ 17	Greater than or equal to	
2 ≤ 10	Less than or equal to	
2 ≠ 7	Not equal to	
≡	Equivalent, equivalence	Used mainly in algebra
2^{10}	Power, index, indices	The 10 is small and above the line of writing
15ºC	Degrees Celsius	The little circle is different to a zero. If temperature then there has to be a letter after the little circle to say which temperature scale. ºC is degrees Celsius, ºF is degrees Fahrenheit, ºK is degrees Kelvin (degrees above absolute zero, H level)
45º	Degrees	Without a letter after it, the little circle stands for the degrees you measure with a protractor, the size of an angle

Whole Numbers

Tables facts

The multiplication square below has the tables facts up to 9 × 9. You will find all of the non-calculator maths, including fractions and ratios, much easier if you learn these facts!

×	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	*7*	8	9
2	2	4	6	8	10	12	*14*	16	18
3	3	6	9	12	15	18	*21*	24	27
4	4	8	12	16	20	24	*28*	32	36
5	5	10	15	20	25	30	*35*	40	45
6	6	12	18	24	30	36	*42*	48	54
7	7	14	21	28	35	42	*49*	56	63
8	*8*	*16*	*24*	*32*	*40*	*48*	*56*	64	72
9	9	18	27	36	45	54	63	72	81

Using the table

7 × 6 = 42

Just go along the 7 row until you reach the 6 column and read off the answer (grey squares)

56 ÷ 8 = 7

Just go along the 8 row until you see 56 and then read off the column number, that is the answer (*italic* numbers)

Some tables are easy to remember

Numbers in the 5 times table (5, 10, 15, 20,...) always ends in 0 or 5, so you know that a number like 37,415 can be divided by 5 without a remainder

Numbers in the 2 times table always have an even last digit

So you know that 31,567,218 can divide by 2 without a remainder

The nine times table has a nice pattern...

9, 18, 27, 36, 45, 54, 63, 72, 81

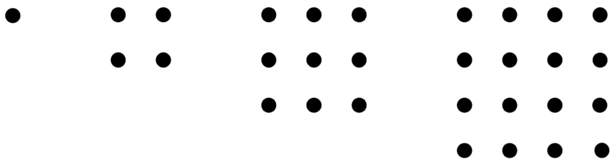
The tens digit goes up by one each time, and the units digit drops by one each time.

If you add the 10s digit and the units digit together, the answer is always 9,
e.g. 45, so 4 + 5 = 9

Challenge: Google 'divisibility tests' for ways of telling if a large number can be divided by any of the numbers up to 9

Square numbers

Look at this pattern of dots...



The number of dots in each square is 1 × 1 = 1, 2 × 2 = 4, 3 × 3 = 9, 4 × 4 = 16 and so on.

So the numbers 1, 4, 9, 16, 25... are called the **square numbers** because you can make them out of square patterns of dots.

Make your own list of square numbers up to 15 × 15

Multiplying a number by itself is a common thing in Maths so there is a special symbol;

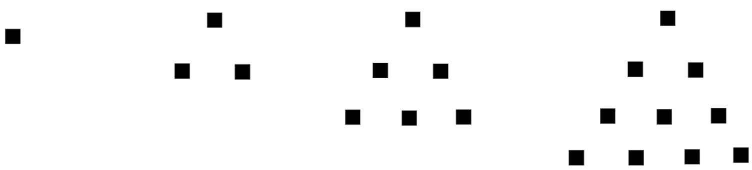
$4 \times 4 = 4^2$

The 2 is small and written above the number.

You read 4^2 aloud as “four squared” or “four to the power two”

See a later section for powers, and 'square roots'

Triangular numbers



The triangle numbers are 1, 3, 6, 10, 15, 21

The triangle numbers are built up using the scheme

1

1 + 2

1 + 2 + 3

1 + 2 + 3 + 4

and so on.

You need to know the name of the pattern

Challenge: Google 'Fibonacci Sequence' and look at some of the images. The *Maths is Fun* page is good with some pictures you can draw on squared paper.

Factors

2 is a factor of 6 because when you divide 6 by 2 there is no remainder.

2 × 3 = 6 so both 2 and 3 are factors of 6.

Factors come in pairs

1 and the number itself are always factors of a number

The best way to find all the factors of a larger number is to work systematically finding pairs of numbers that multiply to give the number.

Example: find all the factors of 24 and list them in order of size

1 × 24 = 24
2 × 12 = 24
3 × 8 = 24
4 × 6 = 24

If you try to carry on, you will find that 5 isn't a factor of 24, and the next pair of numbers is 6 × 4 = 24 which repeats, so you know you have finished and have all the factors of 24.

Just follow the arrow to 'pick off' all the factors in order of size...

1, 2, 3, 4, 6, 8, 12, 24

Square numbers have one repeated factor: Find all the factors of 36

1 × 36 = 36
2 × 18 = 36
3 × 12 = 36
4 × 9 = 36
6 × 6 = 36

All the factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36. You only list the 6 once.
If a number has an odd number of factors, then it *must* be a square number
Challenge: find some numbers that *only* have two factors, so *only* have the top row.

Highest Common Factor (HCF) of two numbers

12 and 6 and 2 are common factors of 24 and 36 because they are factors of both numbers.

To find the highest common factor of 24 and 36, just list out the factors of each number and underline the factors in both lists.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

Then pick the largest one which is 12 in this case.

Challenge: Google 'Cognitive dissonance' in psychology.
The word 'highest' always makes people think of a larger number. The highest common factor of two numbers is (usually) *smaller* than the numbers!

Challenge: Can you find a pair of numbers with an HCF of 1?

Multiples

The multiples of 5 are 5, 10, 15, 20, 25....

The multiples of a number are just the times table for that number

The multiples carry on forever, just keep adding 5 on!

Remember that the *first* multiple of any number is the number itself.

Lowest common multiple (LCM) of two numbers

Example: Find the Lowest Common Multiple of 12 and 16.

List the multiples of 12: 12, 24, 36, 48, 60, 72

List the multiples of 16: 16, 32, 48, 64

Keep going with both lists until you spot the first number that is in both lists...

So the Lowest Common Multiple of 12 and 16 is 48

Cognitive Dissonance again: the word 'Lowest' makes people put down the wrong answer for the LCM. The Lowest Common Multiple of two numbers will (usually) be larger than the two numbers.

Challenge: can you think of two numbers where the LCM is the same as one of the numbers?

Relationship between LCM and HCF of two numbers

The Highest Common Factor of 12 and 16 is 4.

The Lowest Common Multiple of 12 and 16 is 48

12 × 16 = 192

192 ÷ 4 = 48

In general, if A and B are two numbers, the LCM of A and B = The product of A and B divided by the HCF of A and B.

Product means result of multiplying.

In symbols

$$LCM = \frac{A \times B}{HCF}$$

Challenge: choose a few pairs of numbers like 24 and 28 or 15 and 20 and test the relationship between the product, the LCM and the HCF.

Prime numbers and composite numbers

The table below shows the factors of the numbers from 10 to 20

Number	Factors
10	1, 2, 5, 10
11	**1, 11**
12	1, 2, 3, 4, 6, 12
13	**1, 13**
14	1, 2, 7, 14
15	1, 3, 5, 15
16	1, 2, 4, 8, 16
17	**1, 17**
18	1, 2, 3, 6, 9, 18
19	**1, 19**
20	1, 2, 4, 5, 10, 20

These numbers only have two factors, themselves and one They are called the **Prime Numbers**

Prime Numbers have only two factors, the number itself and 1

The other numbers like 10 and 12 and so on are called Composite Numbers.

1 is special. 1 is not a prime number and it isn't a composite number.

Challenge: Make a list of all the prime numbers up to 30.

Challenge: Google 'Sieve of Eratosthenes'. Find a page that explains how to use the sieve to find all the prime numbers up to 100. Try it yourself. Check your list with an animated sieve!

Prime factors of a composite number

$12 = 2 \times 2 \times 3$

You can make any composite number like 12 by multiplying together certain prime numbers.

2, 2 and 3 are called the prime factors of 12.

The prime factors are different from the ordinary factors! Words again!

Each composite number has only one set of prime factors.

$12 = 2^2 \times 3$

Because 12 has 2 twice in its list of prime factors, you can use the power 2 to save writing.

$16 = 2 \times 2 \times 2 \times 2 = 2^4$

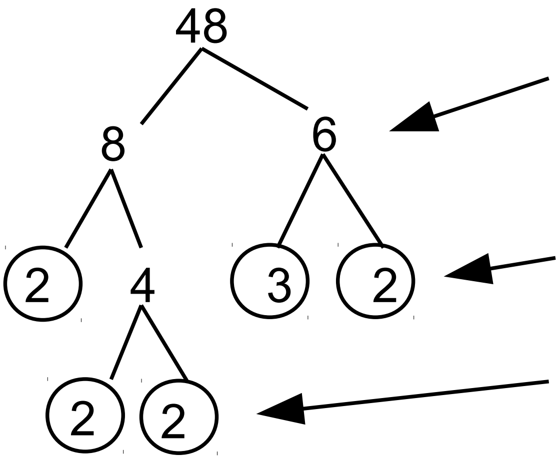
Writing the prime factors of a number with powers is called 'index form'. Look out for that phrase in questions.

You can use a factor tree or repeated division to find the prime factors of a number...

Prime factors using a factor tree

This method relies on you being able to split a number into pairs of ordinary factors.

The factor tree below shows the prime factors of $48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$



Stage 1: Any two numbers that multiply to 48 written here

Stage 2: the 8 and the 6 are both split into factors. Circle the prime numbers.

Stage 3: the 4 is split into 2 × 2.
This is the last stage because all the numbers are prime

Stage 4: pick off all the circled prime numbers. These are the prime factors of 48. You can write them in index form if you need to.

Challenge: re-do the prime factor tree for 48 but using 4 × 12 at the first stage. Then try it again using 2 × 24. You should get the same answers!

Challenge: Google 'interactive factor tree' to find a Web page demo that will draw the tree for you at each stage

Prime factors using repeated division

You can also find the prime factors of a number simply by repeatedly dividing by a list of prime numbers until you reach 1.

Example: find the prime factors of 60.

Start using 2:
60 ÷ 2 = 30 and 30 is even so we can divide by 2 again: 30 ÷ 2= 15
Now 15 is not an even number but it will divide by 3, so 15 ÷ 3 =5
And 5 is a prime number itself, so 5 ÷ 5 = 1
Now make a list of all the prime numbers you have divided by...
2 × 2 × 3 × 5 = 60
You have all the prime factors ($2^2 \times 3 \times 5$ in index form)

Challenge: try finding the prime factors of 48 using the repeated division method

Super challenge: Google "HCF and LCM using prime factors" and see if you can puzzle out how to use the prime factors of two numbers to find the HCF and LCM quickly. This is a higher level topic.

Place Notation

When you write the number 3295, you know that it means 3000 plus 200 plus 90 plus 5 because of the place notation system.

The table below shows the number 3295 written out in columns

Th	H	T	U
3	2	9	5

The 3 digit is worth 3000.

The 9 digit is worth 90.

Numbers in words

The population of the UK was about 63 182 000 in April 2011.

You would say that number in words like this

“sixty-three million, one hundred and eighty two thousand”

Notice the comma after the sixty-three million? We used to put commas in the numbers as well, but now you just use a space.

Suppose we have a few more people born, and the population becomes 63 182 047.

You would say that as

“sixty-three million, one hundred and eighty two thousand and forty seven”

See how the number is broken up into sets of three digits?

And each set of three digits uses the hundreds, tens and units number words?

Look at the grid below, which has the second number written into it.

Millions			**Thousands**			**HTU**		
100M	10M	UM	100T	10T	UT	100	10	U
	6	3	1	8	2	0	4	7

Once you know how to read the numbers up to 999, then you can read any size of number provided you know the millions and the billions.

Examples of numbers up to 999

100	**10**	**U**	**Words**
8	0	6	Eight hundred and six
	2	6	Twenty six
	1	4	Fourteen
1	7	0	One hundred and seventy

Challenge: Google “English number words”. The Wikipedia page has all the words explained with alternatives.

Rounding off to nearest 10, 100, 1000

Th	H	T	U
3	2	9	5

Suppose you want to use the number 3295 in a presentation to an employer.

Perhaps you had 3295 visitors to an exhibition.

You can round the number off to the nearest 1000 by keeping the 3 digit and replacing the rest with 0s.

3295 = 3000 to the *nearest* 1000. “We had over three thousand visitors” is much easier to say in a presentation!

Why 3000 and not 4000? It depends on the 100s digit...

Th	H	T	U
3	2	9	5

Digit we are keeping

'Decider' digit is the one after the digit we keep

If the 'decider' digit is a 0, 1, 2, 3 or 4, then we just replace the decider and all the smaller digits with zeros.

If the 'decider' digit is 5, 6, 7, 8, 9, then we add one to the digit we are keeping and then replace the decider and the smaller digits with zeros.

The 'decider' is always the place after the digit we are keeping.

Examples: 1,763 = 1800 to nearest 100 because we are keeping the hundreds digit, anf the decider is 6, so we add one to the 7. The two digits we are keeping are 1 and 8 but we have to put two zeros in so that the 1 stays in the thousands column and the 8 stays in the hundreds column.

326,492 = 326,000 to nearest 1000 because the 'decider' digit is 4, which means we just replace the decider and all the smaller digits with zeros.

Challenge: Google “rounding using a number line” for another explanation of rounding.

Powers and roots

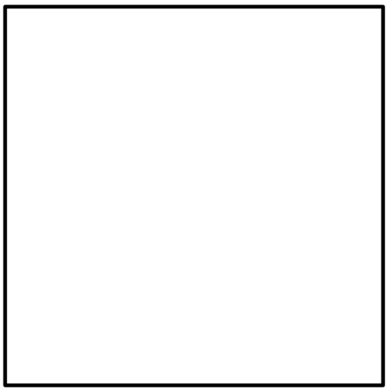
$5 \times 5 = 5^2 = 25$

$5 \times 5 \times 5 = 5^3 = 125$

$5 \times 5 \times 5 \times 5 = 5^4 = 625$

The small digits above the 5 means 'five to the power of'. The small digit is called the 'index' (indices is the plural).

If you draw a square 5cm by 5cm on squared paper, there will be 25 square centimetres inside the square, the shape has an area of 25 cm^2



We call power 2 'squaring' because of this link with area.

Power 3 is called 'cubing' because of the link with volume.

Challenge: get a *lot* of dice. Make a cube out of the dice that measures 2 by 2 by 2. How many dice did you need to make the cube?

Try making a cube with 3 by 3 by 3 dice. Can you calculate how many dice you will need?

Square and cube roots

The 'opposite' of squaring is called 'taking the square root'

The square root of (say) 16 is 4, because $4^2 = 16$

The symbol for taking the square root of (say) 25 would be $\sqrt{25}$

You should find the √ symbol on your calculator. Try it. The answer should be 5.

Another way of looking at the square root is to say “the square root of a number is the side of a square that has the area equal to the number”

Your calculator will tell you that $\sqrt{10}=3.16227766..$

If you draw a square 3.2 cm by 3.2cm with a ruler, the area will be about 10 cm^2

The cube root of 125 is 5 because $5^3 = 125$.

The symbol for the cube root of 125 is $\sqrt[3]{125}=5$. Notice the little 3 before the root symbol. The cube root of a number is the length of the side of a cube that has a volume equal to the number.

Challenge: use a calculator to make a table of squares and cubes of the numbers up to 15. Use your table to find the square root of 144, $\sqrt[3]{2197}$, $\sqrt{64}$, $\sqrt[3]{343}$

Multiplying and dividing powers of the same number

Multiplying powers of the same number

Example: Work out $4^3 \times 4^5$ and leave your answer in index form.

Stage 1: Write out what the symbols mean $4^3 = 4 \times 4 \times 4$ and $4^5 = 4 \times 4 \times 4 \times 4 \times 4$

Stage 2: Write out the symbols in brackets...

$(4 \times 4 \times 4) \times (4 \times 4 \times 4 \times 4 \times 4) = 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$

Stage 3: Write $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$ as a power, 4^8

Stage 4: Notice that $4^3 \times 4^5 = 4^{(3+5)} = 4^8$

The phrase 'leave your answer in index form' means write the answer as a power. Don't work out the value of the number.

Rule: You can multiply powers of the same number by adding the powers.

Dividing powers of the same number

Example: Work out $\frac{3^7}{3^5}$ and write your answer as a power of 3

Stage 1: Write out the powers as a fraction $\frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3}$

Stage 2: Simplify the fraction $\frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3} = \frac{3 \times 3}{1}$

Stage 3: Write the top line as a fraction $3 \times 3 = 3^2$

Stage 4: Notice that $\frac{3^7}{3^5} = 3^{(7-5)} = 3^2$

Rule: You can divide powers of the same number by subtracting the powers

Combined multiplication and division using powers

Example: Write $\frac{7^3 \times 7^8}{7^6}$ as a single power of 7

Stage 1: Use the multiply rule to simplify the top line $\frac{7^3 \times 7^8}{7^6} = \frac{7^{(3+8)}}{7^6} = \frac{7^{11}}{7^6}$

Stage 2: Use the division rule to simply the remaining expression $\frac{7^{11}}{7^6} = 7^{(11-6)} = 7^5$

Challenge: google "GCSE rules of indices" and work through the BBC Bitesize page and associated test.

Negative numbers

Negative numbers are numbers that are less than zero.

Examples include very cold temperatures and overdrafts at the bank.

You can use the scale of a thermometer to reason about negative numbers.

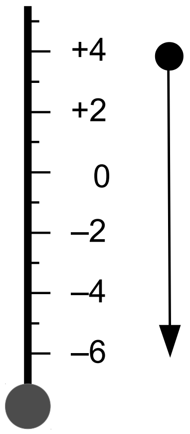
The horizontal number line is often used.

You can extend the rules of arithmetic to deal with adding, subtracting, multiplying and dividing negative numbers.

Negative and positive numbers are sometimes called 'directed numbers' because they have a direction given by their sign.

Number line and temperature

Imagine a thermometer scale like the one below.



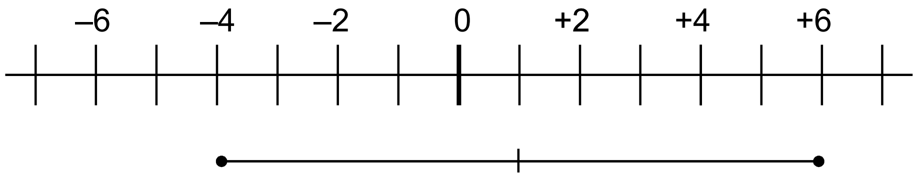
Temperature starts at 4 degrees, then drops by 10 degrees.

The drop is represented by the arrow going down.

The arrow ends at –6 degrees

In symbols, you can write
4 – 10 = –6

A number line is a horizontal scale like the one below with negative numbers on the left and positive numbers on the right.



Example: What number is half way between –4 and 6?
The line is 10 units long, so half way is 5 units positive from –4, which is +1

Adding and subtracting using symbols

Same signs	Add the numbers and give the answer the same sign as the numbers e.g. 4 + 6 = 10 e.g. –20 – 50 = –70
Opposite signs	Find the difference of the numbers and give the answer the sign of the largest number e.g. –30 + 50 = +20 e.g. +3 – 7 = –4
Two minus signs next to each other	"two minuses make a plus" 10 – – 5 = 10 + 5 = 15 One way to think about this is "if the bank cancel a debt, I'm better off" Another way to think about it is "the first minus sign means do the opposite of what follows, and the opposite of subtracting is adding"

Multiplying and dividing using symbols

Suppose three people each owe you £5. Together they owe you £15.

That story suggests that 3 × –5 = –15

If you know the signs of the numbers you are multiplying, then you know the sign of the answer.

Same signs	Positive answer –4 × –8 = +32 –100 ÷ –20 = +5 +3 × +4 = 12 (could be written 3 × 4 = 12)
Opposite signs	Negative answer –5 × 6 = –30 20 ÷ –5 = –4

Challenge: Google "GCSE directed numbers" and try some on-line practice exercises.

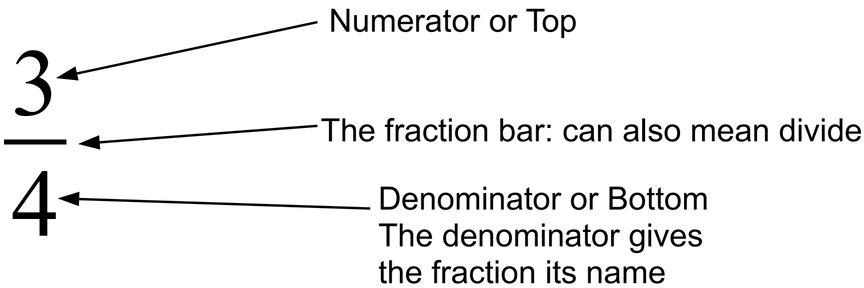
Fractions

Proportional arithmetic is important in everyday life.

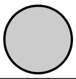
Scaling up recipes, diluting medicine and squash, mixing mortar from cement and sand and water, dividing up costs *in proportion* to (say) the office space each department uses.

Foundation GCSE Maths has questions that use visual representations of fractions and proportions as well as arithmetic based approaches.

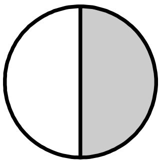
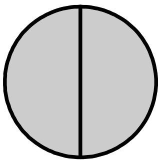
Fraction words



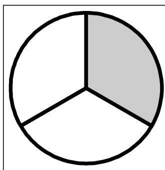
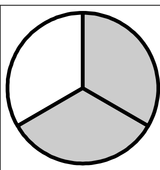
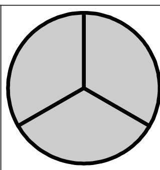
The English words for the common 'fraction families' are shown below.

$\frac{1}{1}=\frac{2}{2}=\frac{3}{3}...$	One or 'whole'	

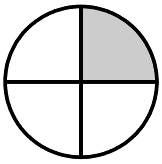
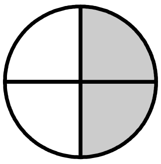
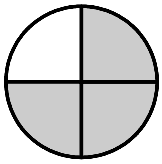
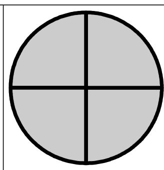
Half / Halves

$\frac{1}{2}$	"One half" or just "half" Decimal: 0.5 Percentage: 50%	
$\frac{2}{2}$	"Two halves make a whole"	

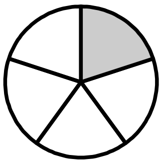
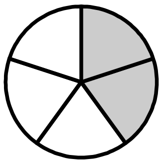
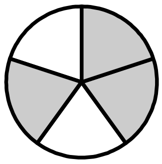
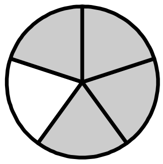
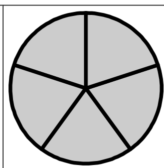
Third / Thirds

 $\frac{1}{3}$	 $\frac{2}{3}$	 $\frac{3}{3}$
"One third" or just "a third"	"Two thirds"	"Three thirds make a whole"
Decimal 0.3333... ≈ 0.33 Percentage = 33.33333...%	Decimal 0.666... ≈ 0.67 Percentage 66.66...%	Decimal = 1.0 Percentage = 100%

Quarter / quarters

 $\frac{1}{4}$	 $\frac{2}{4}$	 $\frac{3}{4}$	 $\frac{4}{4}$
“One quarter” or just “a quarter”	“Two quarters” which is the same as a half	“Three quarters”	“Four quarters” which is the same as a whole
Decimal 0.25 Percentage 25%	Decimal 0.5 Percentage 50%	Decimal 0.75 Percentage 75%	Decimal = 1.0 Percentage = 100%

Fifth / fifths

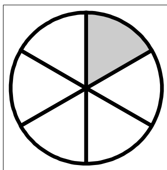
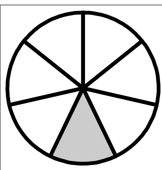
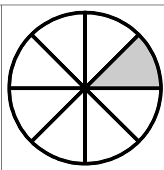
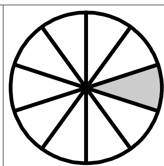
				
“One fifth” or just “a fifth”	“Two fifths”	“Three fifths”	“Four fifths”	“Five fifths” which is the same as a whole
Decimal 0.2 Percentage 20%	Decimal 0.4 Percentage 40%	Decimal 0.6 Percentage 60%	Decimal 0.8 Percentage 80%	Decimal = 1.0 Percentage = 100%

Fractions with larger denominators (smaller parts)

For denominators higher than 4, the names are simply the number word with 'th' on the end.

The only 'special' fraction names are half, third, quarter, because those are the ones we use most often.

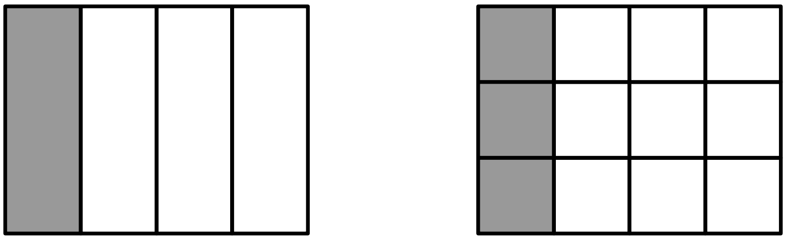
You might see 'fourth' instead of 'quarter' sometimes.

			
“Sixths”	“Sevenths”	“Eighths”	“Tenths”

Challenge: Google “definition of fraction” to find out where the word came from.

Equivalent fractions and simplifying

Look at the rectangles below. What fraction is shaded?

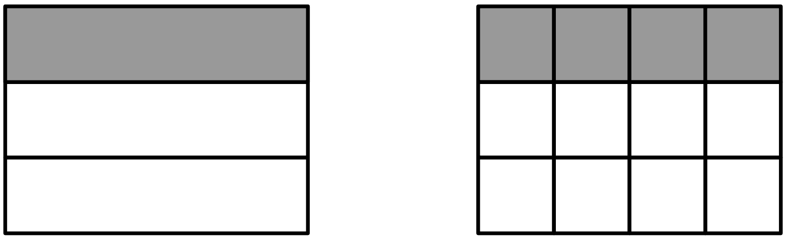


$\frac{1}{4}$ is shaded in the first diagram and $\frac{3}{12}$ is shaded in the second diagram.

The rectangles are the same size, and the shaded areas are the same, so $\frac{1}{4}=\frac{3}{12}$

These fractions are called **equivalent**.

Look at *another* set of rectangles below



These rectangles show the **equivalence** of $\frac{1}{3}$ and $\frac{4}{12}$

Notice how both $\frac{1}{3}$ and $\frac{1}{4}$ can be shaded in using a rectangle with 12 equal squares.

Challenge: make a set of rectangles that you could use to show that $\frac{1}{5}=\frac{4}{20}$. Make another set of rectangles to show that $\frac{1}{4}=\frac{5}{20}$

Writing fractions over new bottom

$\frac{3}{4}=\frac{?}{20}$

How do you find the missing number?

Visual method: draw a rectangle of 5 by 4 squares and colour in $\frac{3}{4}$ of the squares.

Arithmetical method: 4 goes into 20 five times, so I have to multiply the top by five, so I have $\frac{15}{20}$

Challenge: Google “Equivalent Fractions” and look for the Maths Is Fun page. Try the questions.

Simplifying fractions

Question: Simplify $\frac{12}{20}$

What they want you to do: divide the top and bottom of the fraction by the same number until the top and bottom don't have any common factors any more.

Example: $\frac{12}{20}=\frac{12\div 2}{20\div 2}=\frac{6}{10}$ and $\frac{6\div 2}{10\div 2}=\frac{3}{5}$

Simplifying is also called 'cancelling down' or 'writing in lowest terms' in some books.

Another example that isn't just 2 all the time: $\frac{12}{18}=\frac{12\div 2}{18\div 2}=\frac{6}{9}$ and $\frac{6\div 3}{9\div 3}=\frac{2}{3}$

Challenge: Google “Simplifying fractions” and try the questions on the Maths Is Fun page.

Super mega challenge: You can simplify any fraction by finding the **prime factors** of the top and the bottom and crossing corresponding pairs out.

For instance $\frac{72}{96}=\frac{2\times 2\times 2\times 3\times 3}{2\times 2\times 2\times 2\times 2\times 3}$ so we can 'cross off' three of the 2s and one 3 on both the top and bottom.

You get get the prime factors of the simplified fraction: $\frac{3}{2\times 2}=\frac{3}{4}$

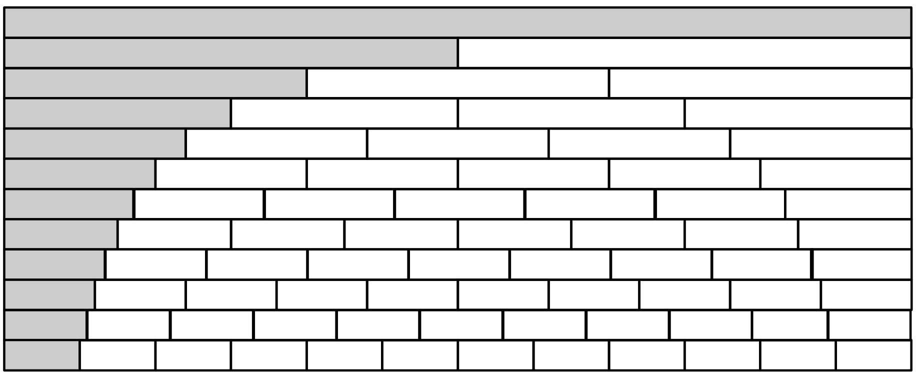
Use this prime factor method to simplify the fraction $\frac{1911}{5005}$

Check your answer by typing “simplify 1911/5005” into the search box at wolframalpha.com

Fraction wall

A fraction wall is a good visual way of spotting equivalent fractions.

It is the *length* of the rows in the wall that matter here!



The grey sections of each row in the wall represent this sequence of fractions...

1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{11}$, $\frac{1}{12}$

Can you see how the $\frac{1}{3}$ shaded section is the same length as *two* of the sixths?

Challenge: make your own fraction wall and label it.

Putting fractions in order of size

Ordering unit fractions

Fractions with numerator (top) 1 are called 'unit fractions' $\frac{1}{5}$, $\frac{1}{17}$, $\frac{1}{2}$, $\frac{1}{4}$

As the denominator (bottom) gets larger, the value of the fraction gets smaller.

You can easily sort these fractions into size order.

The fractions with the *largest* denominator have the *smallest* value.

You can think of it as the whole cake being shared out between more people.

Example: Put these fractions in ascending order $\frac{1}{5}$, $\frac{1}{17}$, $\frac{1}{2}$, $\frac{1}{4}$

'Ascending' order means the list starts with the smallest value and increases.

The correct order is $\frac{1}{17}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{2}$

Ordering fractions with the same denominator

Example: Put these fractions in order from **largest** to **smallest**. $\frac{4}{13}$, $\frac{7}{13}$, $\frac{2}{13}$, $\frac{9}{13}$

Because the denominators are all the same, you just write the fractions in order of the numerator. It is like counting slices of pizza.

The correct order is $\frac{9}{13}$, $\frac{7}{13}$, $\frac{4}{13}$, $\frac{2}{13}$

Ordering general fractions

When the denominators are all different, and the numerators are not all 1, you just have to find equivalent fractions that all have the same denominator.

This involves...

1) Finding a common denominator for all the fractions
2) Writing each fraction over the common denominator
3) Working out the correct order by comparing numerators
4) Writing the original fractions out in the correct order

Example: put these fractions in order of size, **smallest** to **largest** $\frac{4}{5}$, $\frac{7}{10}$, $\frac{5}{6}$, $\frac{1}{2}$

Find the common denominator	2 and 5 are both factors of 10, so focus on 10 and 6. LCM of 10 and 6 is 30, so use 30 as the common denominator.
Write each fraction over the common denominator	$\frac{4}{5}=\frac{24}{30}$, $\frac{7}{10}=\frac{21}{30}$, $\frac{5}{6}=\frac{25}{30}$, $\frac{1}{2}=\frac{15}{30}$
Work out the correct order	$\frac{1}{2}=\frac{15}{30}$, $\frac{7}{10}=\frac{21}{30}$, $\frac{4}{5}=\frac{24}{30}$, $\frac{5}{6}=\frac{25}{30}$
Write the original fractions in the correct order	$\frac{1}{2}$, $\frac{7}{10}$, $\frac{4}{5}$, $\frac{5}{6}$

Challenge: Google “comparing fractions” and try the questions on the Maths Is Fun page for saying which fraction is the largest out of a pair of fractions.

Adding and subtracting fractions

What is adding?

When you add 5 to 7 what do you actually do?

Most people will break the 5 into a 3 and a 2. The 3 and the 7 make 10 and then the 2 is the units digit, so 5 + 7 = 12.

Some people 'count on' from 7: 7, 8, 9, 10, 11, 12

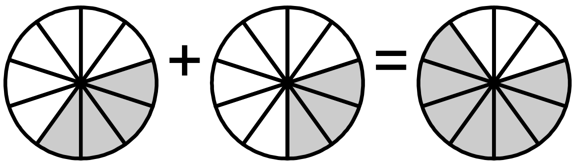
You can count on with fractions, but *only* if the fractions have the same denominators.

You can subtract by counting back, but again if the fractions have the same denominators.

If the fractions have different denominators, you have to find a common denominator before you can add or subtract them.

Same denominators: count the slices

$$\frac{4}{10} + \frac{3}{10} = \frac{7}{10}$$

In pictures:  (count those slices!)

Notice how the **denominator does not change**.

If you think about 'counting the slices' you will avoid any temptation to write this:

$\frac{1}{7}+\frac{3}{7}=\frac{4}{14}$ (WRONG!)

Subtracting fractions with the same denominator works in the same way: just subtract the tops of the fractions.

$$\frac{11}{12}-\frac{5}{12}=\frac{6}{12}$$

Notice that $\frac{6}{12}$ *simplifies* to $\frac{1}{2}$.

Different denominators

Example: $\frac{1}{3}+\frac{2}{5}$

We can't add these fractions directly.

Instead we need to write the fractions over a common denominator.

Once the fractions are over the same denominator, you can just add the tops.

The lowest common multiple of 3 and 5 is 15, so we'll use 15 as the denominator.

$\frac{1}{3}=\frac{5}{15}$ and $\frac{2}{5}=\frac{6}{15}$ (can you remember how to find equivalent fractions?

Then you just add the fractions over 15 by adding the tops

$$\frac{5}{15}+\frac{6}{15}=\frac{11}{15}$$

You can't simplify the $\frac{11}{15}$ and this is related to the fact that 3 and 5 have no common factors (except 1).

Challenge: Try adding $\frac{3}{8}+\frac{1}{4}$. You only have to change one of the fractions.

Subtracting works in the same way. Write the fractions over the same denominator and then subtract the tops.

Example: $\frac{6}{10}-\frac{7}{15}$

The lowest common multiple of 10 and 15 is 30, so we can use that as the common denominator.

Writing the fractions over the common denominator: $\frac{6}{10}=\frac{18}{30}$ and $\frac{7}{15}=\frac{14}{30}$

Subtracting the tops: $\frac{18}{30}-\frac{14}{30}=\frac{4}{30}$

But $\frac{4}{30}$ simplifies to $\frac{2}{15}$

Challenge: Try working out $\frac{2}{3}-\frac{4}{9}$. The common denominator is 9.

Multiplying fractions

Example: $\frac{1}{3}\times\frac{4}{5}$

You multiply the tops to get the new top

You multiply the bottoms to get the new bottom

$\frac{1}{3}\times\frac{4}{5}=\frac{1\times4}{3\times5}=\frac{4}{15}$

Multiply means 'of' so you can read $\frac{1}{3}\times\frac{4}{5}$ as 'take one third of four fifths'.

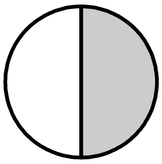
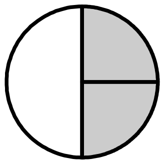
Example: $\frac{3}{10}\times\frac{1}{6}$.

You can save time by 'cancelling' the 3 and the 6 to change the calculation to $\frac{1}{10}\times\frac{1}{2}$

Challenge: Google “multiplying fractions by cancelling down” and try the BBC Bitesize page about multiplying fractions.

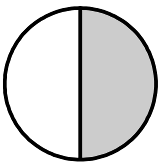
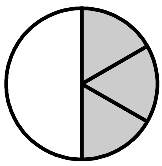
Visual explanation of multiplying

Most people can remember that a **half of a half** is a quarter!

Visually:  split the shaded side into two  and each bit is a quarter!

In fractions, taking half of a half means $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

You can use pizzas to work out **one third of a half**. Below is half a pizza shaded in

 split the shaded half into three  each bit is one sixth!

you have one sixth! So a third of a half is one sixth. In symbols $\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$

When you multiply fractions less than one, the answer gets smaller!

Multiplying a fraction by a whole number

Whole numbers can be represented by fractions with denominator 1.

Example: $5\times\frac{3}{4}$

Just write as $\frac{5}{1}\times\frac{3}{10}$ then multiply tops and bottoms $\frac{5}{1}\times\frac{3}{20}=\frac{15}{20}=\frac{3}{4}$

Dividing fractions

Dividing by a unit fraction

How many threes are there in 12?

12 ÷ 3 = 4

When you ask a question about 'how many?' the answer usually involves a division.

Suppose you have three whole pizzas. **How many** quarter pizza slices can you cut from them?

You have three pizzas, and each has 4 slices, so you get 12 slices all together.

The 'how many' part looks like this: $3 \div \frac{1}{4}$ but the answer looks like 3 × 4 = 12.

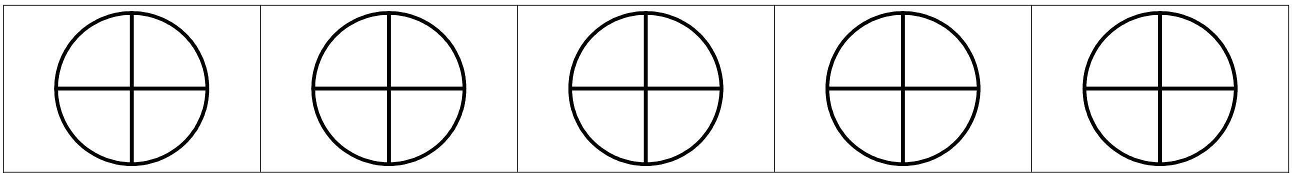
So *dividing* by a unit fraction is like *multiplying by the bottom*.

Example: $7 \div \frac{1}{3} = 7 \times 3 = 21$

Dividing by fractions with tops larger than 1: visual

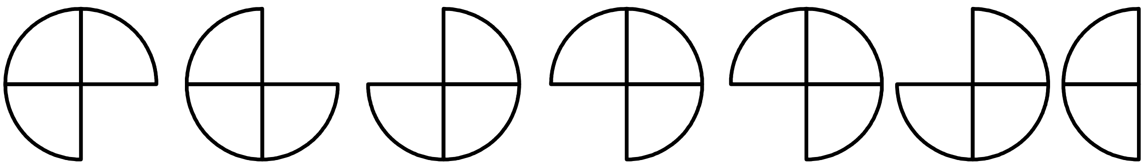
Example: $5 \div \frac{3}{4}$

We are asking 'how many $\frac{3}{4}$ are there in 5'?



There are 20 quarters in 5 whole ones

If we split the 20 quarters up and then re-group them into threes, we get



So we have 6 groups of $\frac{3}{4}$ with two quarters left out. The tricky bit is the two quarters that are left out. They make two-*thirds* of $\frac{3}{4}$.

So $5 \div \frac{3}{4} = 6\frac{2}{3}$

Challenge: use a visual method like this to work out $4 \div \frac{2}{3}$

Dividing fractions: arithmetic method

Example: $5 \div \frac{3}{4}$ lets do this calculation using just arithmetic

Remember that in the visual approach, you split up the 5 into 20 quarters, so you *multiplied* by 4.

Then you grouped the 'floating' quarters into groups of 3. You *divided* by 3.

So in arithmetic terms, the calculation looks like this $5 \div \frac{3}{4} = \frac{5}{1} \times \frac{4}{3} = \frac{5 \times 4}{1 \times 3} = \frac{20}{3}$

You can write 20 thirds as 6 whole ones and two thirds (see below) $\frac{20}{3} = 6\frac{2}{3}$

The 'rules' for the arithmetic method are

- Turn the fraction you are dividing by upside down
- Then multiply the fractions!
- Simplify if you can
- Write as a mixed number (see below)

You can use these rules when both of the numbers are fractions

Example: $\frac{4}{5} \div \frac{2}{9}$. Just apply the rules

Turn the second fraction upside down and change to multiply: $\frac{4}{5} \times \frac{9}{2}$

So $\frac{4}{5} \times \frac{9}{2} = \frac{36}{10}$. Simplifying $\frac{36}{10}$ gives $\frac{18}{5}$ which is the same as $3\frac{3}{5}$

Challenge: Google “Multiplying fractions”. Try the Maths Is Fun page.

See below for the meaning of the word 'reciprocal' which is used on that page.

Reciprocal

The reciprocal of a number is one divided by the number.

Example: The reciprocal of 3 is $\frac{1}{3}$.

Example: How do you find the reciprocal of $\frac{2}{5}$?

Just divide 1 by the fraction: $1 \div \frac{2}{5} = \frac{1}{1} \times \frac{5}{2} = \frac{5}{2}$ Writing $\frac{5}{2}$ as a mixed number $2\frac{1}{2}$

Mixed numbers: fractions larger than 1

All the fraction methods so far can be extended to mixed numbers.

A mixed number is a number that has a combination of whole numbers and fractions.

Example: $2\frac{1}{2}$ is a mixed number. It means 2 whole ones plus $\frac{1}{2}$

Strange but true: Mixed numbers are the only place in the whole of maths where writing two symbols next to each means add. See Algebra.

Fraction Words: Improper / top heavy fractions

Type of fraction	Official words	What they mean
$\frac{3}{5}$	Proper fraction	Top is less than bottom, so value is less than 1
$\frac{20}{3}$	Improper fraction	Top is larger than bottom, so value is more than one. Known as 'top heavy' when teaching! You can find out how many whole ones there are by dividing the top by the bottom.
$6\frac{2}{3}$	Mixed number (some textbooks use 'mixed fraction')	A whole number plus a fractional part

Vulgar fractions: Some textbooks will refer to proper and improper fractions as 'vulgar fractions'. A vulgar fraction just has a numerator and a denominator, no whole number.

Converting between mixed numbers and improper fractions

The arithmetical methods for dealing with fractions work best with fractions with just a top and a bottom. You need to be able to change between mixed numbers and improper fractions.

Improper (top heavy) to mixed

When you finish a multiplication sum with fractions, you often end up with a top heavy fraction. You need to convert it to a mixed number.

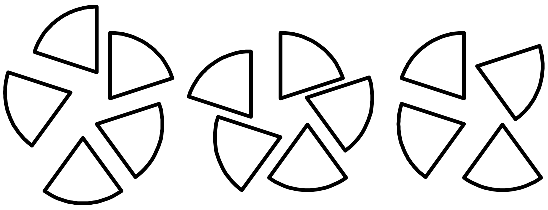
Example: Convert $\frac{14}{5}$ to a mixed number.

- Divide the bottom into the top to find the whole number. 5 goes into 14 two times so the whole number is 2
- Put the remainder over the bottom to find the fractional part. Remainder is 4, so the fraction is $\frac{4}{5}$

So you have $\frac{14}{5}=2\frac{4}{5}$

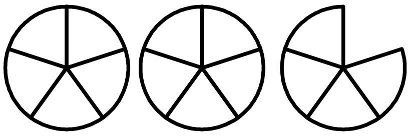
Top heavy to mixed using pizza slices

Messy party. Lots of pizza left over. Look at all these slices! 14 of them.



Five slices make a whole pizza. Written as a fraction $\frac{14}{5}$

Arranging the slices back on the plates gives this...



So we have two whole pizzas and 4 fifths of a pizza left.

Written as a mixed number $2\frac{4}{5}$

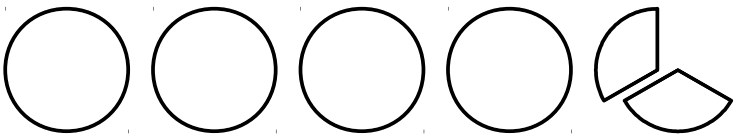
Mixed to improper (top heavy)

Example: convert $4\frac{2}{3}$ to an improper fraction (top heavy)

- Multiply the denominator by the whole number: 3 × 4 = 12
- Add the numerator of the fraction part: 12 + 2 = 14
- Put the result over the denominator: $\frac{14}{3}$

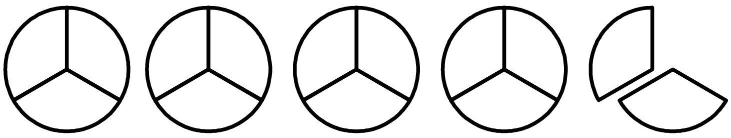
Mixed to improper visual way

Example: convert $4\frac{2}{3}$



Four whole pizzas and two one third size slices.

Everyone wants the slices to be the same size, so we slice the whole pizzas into 3



Just count up all the slices, number them. You should end up with 14. So $4\frac{2}{3}=\frac{14}{3}$

Challenge: Google "Convert between improper fractions and mixed numbers" and try the Maths Is Fun page with the questions at the bottom.

Adding and subtracting mixed numbers

You can add mixed numbers by working with the wholes and with the fractions separately, then adjusting the wholes if the fractional parts add up to more than one.

You can subtract mixed numbers in just the same way, but 'crossing off' may be involved. Depending on the question, it might be easier to turn all the numbers top heavy to start off with.

As always, with adding and subtracting fractions, you will need to use common denominators.

Adding mixed numbers by adding wholes then fractions

Example: $2\frac{7}{8}+3\frac{1}{4}$

Stage 1: Lets split the problem into wholes and fractions. $2+3+\frac{7}{8}+\frac{1}{4}$

Stage 2: Add the wholes 2 + 3 = 5 and record that result

Stage 3 Add the fractions $\frac{7}{8}+\frac{1}{4}$

Common denominator: 4 is a factor of 8 so we can use 8 as the denominator and just change the quarter into eighths $\frac{1}{4}=\frac{2}{8}$ so the fraction sum becomes $\frac{7}{8}+\frac{2}{8}=\frac{9}{8}$

But $\frac{9}{8}$ is top heavy, so we can write it as a mixed number $1\frac{1}{8}$

Stage 4: Now we have to finish the calculation by adding the wholes and the mixed number $5+1\frac{1}{8}=6\frac{1}{8}$

Challenge: find an exercise in a GCSE textbook try adding some mixed numbers this way. Check your answers. Doing this revises most of the fractions part of the syllabus!

Calculator challenge: if you have a scientific calculator with fractions keys find out how to use them.

Easy subtracting

Compare the fractional parts of the mixed numbers. If the fractional part of the first mixed number is larger than the fractional part of the second number, this method will work fine and quickly.

Example: $7\frac{5}{8}-3\frac{2}{5}$

$\frac{5}{8}$ is larger than $\frac{2}{5}$ because $\frac{5}{8}$ is over half and $\frac{2}{5}$ is less than half.

So you can take the $\frac{2}{5}$ away from the $\frac{5}{8}$ without any 'crossing off' or 'borrowing' and can work with the whole numbers and fractions separately.

Stage 1: $7\frac{5}{8}-3\frac{2}{5}$ is 7 – 3 and $\frac{5}{8}-\frac{2}{5}$

Stage 2: 7 – 3 = 4

Stage 3: $\frac{5}{8}-\frac{2}{5}=\frac{25}{40}-\frac{16}{40}=\frac{9}{40}$

Stage 4: $4+\frac{9}{40}=4\frac{9}{40}$

Subtracting mixed numbers with 'crossing off'

This method is for when you can't subtract the fractional parts separately from the whole number parts. Its like 'crossing off'.

Example: $6\frac{3}{5}-3\frac{7}{10}$

Stage 1: Comparing the fraction parts: $\frac{3}{5}$ is smaller than $\frac{7}{10}$ so we can't just separate the whole and the fraction parts and subtract them separately.

Instead we 'cash in' one of the whole ones for 5ths like this

$5+1\frac{3}{5}-3\frac{7}{10}$ then we turn the $1\frac{3}{5}$ top heavy like this $5+\frac{8}{5}-3\frac{7}{10}$

Now you can subtract the whole numbers and subtract the fractions as before

Stage 2: 5 – 3 = 2 whole ones

Stage 3: $\frac{8}{5}-\frac{7}{10}$ use common denominator 10, $\frac{16}{10}-\frac{7}{10}=\frac{9}{10}$

Stage 4: Putting the whole and the fraction together so the final answer is a mixed number: $2\frac{9}{10}$

Subtracting mixed numbers by making them all top heavy

It is sometimes easier to just turn all the fractions top heavy to avoid 'crossing off'.

Example: $2\frac{3}{4}-\frac{7}{8}$

Stage 1: Just make the $2\frac{3}{4}$ top heavy, so the calculation turns into $\frac{11}{4}-\frac{7}{8}$

Stage 2: Use common denominator of 8, calculation becomes $\frac{22}{8}-\frac{7}{8}$

Stage 3: Answer is $\frac{22}{8}-\frac{7}{8}=\frac{15}{8}$

Stage 4: Convert to mixed number $\frac{15}{8}=1\frac{7}{8}$

Exam note: If the question has mixed numbers, it is a good idea to write any top heavy fractions in the answer as mixed numbers!

Multiplying and dividing mixed numbers

Golden rule: make all the mixed numbers top heavy!

Why?: Because a mixed number like $2\frac{3}{4}$ is really $2+\frac{3}{4}$ and you have to put the addition in a bracket to multiply by something else (See BIDMAS section in Algebra).

It is easier to get rid of the addition by turning $2\frac{3}{4}$ into a top heavy fraction $\frac{11}{4}$.

Multiplying a mixed and proper fraction

Example: $2\frac{3}{4}\times\frac{2}{3}$

Stage 1: Turn top heavy $\frac{11}{4}\times\frac{2}{3}$

Stage 2: Do the multiplying $\frac{11}{4}\times\frac{2}{3}=\frac{11\times2}{4\times3}=\frac{22}{12}$ (you could have cancelled the 2 and 4)

Stage 3: Check for any simplification $\frac{22}{12}=\frac{11}{6}$

Stage 4: Turn any top heavy answers back to mixed numbers $\frac{11}{6}=1\frac{5}{6}$

Challenge: find an exercise in a GCSE Maths text book, try to work out all the answers and check if you are getting them right. Ask for help if you find you get a lot wrong!

Dividing a mixed number by a proper fraction

Example: $7\frac{3}{4} \div \frac{3}{5}$

Stage 1: Turn any mixed numbers top heavy $7\frac{3}{4} = \frac{31}{4}$

Stage 2: Turn the fraction you are dividing by upside down and change the divide to a multiply $\frac{31}{4} \times \frac{5}{3}$

Stage 3: Multiply tops and multiply bottoms $\frac{31}{4} \times \frac{5}{3} = \frac{31 \times 5}{4 \times 3} = \frac{155}{12}$

Stage 4: If answer top heavy then convert it to a mixed number $\frac{155}{12} = 12\frac{11}{12}$

Common sense check: we are asking how many times $\frac{3}{5}$ fits inside $7\frac{3}{4}$. Well, $\frac{3}{5}$ is just a bit over half, and $7\frac{3}{4}$ is nearly 8, so we would expect an answer a bit less than 16. Our actual answer is nearly 13, so that seems about right.

Challenge: find an exercise in a GCSE Maths text book, try to work out all the answers and check if you are getting them right. Ask for help if you find you get a lot wrong!

Decimals

What are decimal numbers?

Next time you go to the supermarket, save the till receipt. There is a lot of everyday maths behind a till receipt.

		£
00930314	PERCY BAG	0.10
00392990	OLIVE ROLLS	1.69<
00852180	NEW POTS 1.1KG	1.79<
00797016	NUTTY W/FD SLD	2.00<
00869041	APP/CEL/WNUT SL	2.00<
00975568	UK GARDEN PEAS	1.99<
00392990	OLIVE ROLLS	1.69<
Balance before Saving		11.26
**Bakers Choice 2 fr £2.50		-0.88
Produce 2 for £3		-1.78
Items: 7	Balance to Pay	8.60
Cash Tendered		10.00
Cash Due		1.40

****
You Have Saved £2.66 On
Our Promotions Today
****

The costs are always written to **two decimal places** of pounds**.**

The **first decimal place** is the number of 10p pieces in the cost.

The **second decimal place** is the number of 1p pieces in the cost.

You get 10 ten pence pieces in a pound so a 10p is $\frac{1}{10}$ of a pound,

there are 100 one pence pieces so 1p is $\frac{1}{100}$ of a pound.

My new potatoes cost £1.79 which means $1+\frac{7}{10}+\frac{9}{100}$ of a pound written in fractions.

Decimal numbers are a convenient way of talking about fractions of quantities.

Continuing the place notation system to 'places' less than one makes arithmetic a lot easier, as you can add and subtract in columns just like whole numbers.

The decimal places carry on forever after the decimal point. The grid below shows a number with **three decimal places**.

Th	H	T	U	●	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
3	2	9	5	●	6	1	7

Challenge: Google "Powers of 10" and look for the Powers of Ten Web site. Explore the site.

Rounding off to decimal places

Suppose the number 14.31502 is displayed on your calculator.

Suppose you have just worked out an amount of money in pounds.

You need to round the calculator display to **two decimal places**.

Putting the number in a set of columns might help

'decider' is the 3^{rd} decimal place

Ordinary columns					Decimal places				
					1st	2nd	3rd	4th	5th
Th	H	T	U	●	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$	$\frac{1}{100000}$
		1	4	●	3	1	5	0	2

We need to keep the first two decimal places.

The 3^{rd} decimal place tells us if we have to round up or chop the number off. I call this the 'decider' place and it is always one after the smallest place we are keeping.

If the digit in the 3^{rd} decimal place is 5, 6, 7, 8, 9 we add one to the 2^{nd} decimal place

If the digit in the 3^{rd} decimal place is 0, 1, 2, 3, 4 we just chop the number off

In this case, we round *up*, and write the answer as £14.32

Challenge: see if you can follow the reasons for the rounding in the table below...

Calculator display	**Rounded off number (all 2dp)**	**Explanation**
0.57499999999	0.57	'decider' digit in 3^{rd} decimal place is 4, so chop off. The 9s don't make any difference
23.7878787878	23.79	The 'decider' is 5 or more so round up by adding one to the digit in 2^{nd} decimal place.
6.29537231487	6.30	The decider in the 3^{rd} decimal place is 5 so you have to add one to the 9 in the second place, so you carry 1. You *must* put the zero on the end to make the number 2dp
0.06391945823	0.06	The 'decider' is 3 so chop off the number after 2dp. It is just small!
16.0049999999	16.00	'Decider' says chop off. Two zeros kept to say the number is 2dp
16.005	16.01	'Decider' says to add 1 to 2^{nd} dp.

Rounding off to significant figures

The first significant figure in a number is the first digit on the left that is not a zero.
We commonly round off results of calculations that are not about money to 3 significant figures.
It is easier to look at numbers much larger than 1 separately from numbers much smaller than 1.

Numbers larger than 1

The example below has a decimal number which has a 2 in the thousands column so that is the first significant figure.

Significant figures (sf)									
1st	2nd	3rd	4th		5th	6th	7th	8th	9th
Th	H	T	U	●	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$	$\frac{1}{100000}$
2	9	3	7	●	6	1	5	8	2

Rounding this off to 3 significant figures means you want to keep the 2, 9 and 3 digits and that the 'decider' is the digit in the 4^{th} significant figure, in this case 7.

The 'decider' is 5 or more so you add one to the three, so now the number looks like this...

Significant figures (sf)									
1st	2nd	3rd	4th		5th	6th	7th	8th	9th
Th	H	T	U	●	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$	$\frac{1}{100000}$
2	9	4		●					

But we want to show that the 2 is in the thousands column, the 9 is in the hundreds column and the 4 is in the tens column without having to write out the columns.

We have to put a zero in the Units column, even though that is not a significant figure!

Significant figures (sf)									
1st	2nd	3rd	4th		5th	6th	7th	8th	9th
Th	H	T	U	●	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$	$\frac{1}{100000}$
2	9	4	0	●					

The rounded number is written as 2940 and you don't write a decimal point or *any* figures after the decimal point.

Challenge: see if you can follow the reasons for the rounding for each number in the table below.

Calculator display	**Rounded number (all to <u>three</u> significant figures)**	**Explanation**
2 647 321	2 650 000	7 says to add one to the 4 but we have to add zeros to keep the 2 in the millions column
2013	2010	1 does not change, but the 4^{th} significant figure replaced by zero to keep the 2 in the thousands column
175.8418	176	The 'decider' is the 4^{th} significant fugure which is 8, so we add one to the 3^{rd} significant figure. No need for extra zeros because we have the 6 in the Units
2.45823917	2.46	Again a round up. No need for extra zeros.
4296.7199	4300	4^{th} sf is 6 so add one to 3^{rd} sf. Carry one to 2^{nd} sf. Two zeros needed to keep 4 in thousands and 3 in hundreds.

Numbers smaller than 1

Example: round 0.006768129 to two significant figures.

Significant figures (sf)										
	●			1st	2nd	3rd	4th	5th	6th	7th
0	●	0	0	6	7	5	8	1	2	9

The first significant figure is the 6 in the $\frac{1}{1000}$ column.

The two zeros between the decimal point and the 6 are very important because they keep the following digits in the right columns. But they are not 'significant'.

We keep the 6 and the 7 digit and check the 'decider' which is 8, so you have to add one to the 7 to round up.

The rounded number becomes

Significant figures (sf)										
	●			1st	2nd	3rd	4th	5th	6th	7th
0	●	0	0	6	8					

You don't put zeros after the 2^{nd} significant figure. Just don't write anything.

So 0.006768129 ≈ 0.0068 (2sf)

Challenge: see if you can follow the reasons for the rounding for each number in the table below.

Calculator display	Rounded number (all to <u>two</u> significant figures)	Explanation
0.927239123	0.93	Round up because of the 7 in 3rd sf
0.000341297	0.00034	Chop off because the 3rd sf is 1
0.08959918	0.090	The 5 in 3rd sf says to add one to the 2nd sf which is a 9, so carry 1 to 1st sf. Need to keep the trailing zero to show 2sf
0.9951285	1.0	Extreme example. 5 in 3rd sf says to add 1 to the 9 in the 2nd sf but then you have lots of carries. Keep *one* trailing zero to show 2sf
0.02069124	0.021	The 6 in the 3rd sf says to add one to the 0 in the 2nd sf

Rounding to one significant figure

Rounding numbers to one significant figure is handy in estimating (see the BIDMAS topic later).

The results can be a bit surprising.

Challenge: see if you can follow the reasons for the rounding for each number in the table below.

Calculator display	Rounded number (all to <u>one</u> significant figure)	Explanation
53 012 456	50 000 000	Just a lot of zeros needed!
45	50	Round up
19 950 000	20 000 000	The add one to the nine and carry situation
3.000937	3	Don't be fooled by decimal points
0.0085001289	0.009	Round up and keep the zeros after dp

As a final thought about rounding, remember that

- 49 rounded to the nearest hundred = 0 (!) but 50 to the nearest hundred = 100
- the most significant numbers must stay in the same column, rounding can't multiply or divide a number

Challenge: Google “Rounding Numbers” and try reading the Maths Is Fun page and doing the questions at the bottom.

Converting decimals to fractions

Example: write 0.85 as a fraction in its simplest form

Remember that 0.85 = $\frac{8}{10}+\frac{5}{100}$

You can add the fractions using a common denominator of 100

$$\frac{8}{10}+\frac{5}{100}=\frac{80}{100}+\frac{5}{100}=\frac{85}{100}$$

And $\frac{85}{100}$ simplifies to $\frac{17}{20}$

You can just count the number of decimal places and use that to find the denominator.

A number with 1 decimal place has 10 as the denominator, a number with 2 decimal places has 100 as denominator and a number with three decimal places has 1000 as the denominator.

Example: write 0.625 as a fraction in its simplest form

Stage 1: count the decimal places and decide the denominator to use.
0.625 has 3 decimal places so use 1000 as the denominator

Stage 2: put the digits after the decimal point over the chosen denominator
$\frac{625}{1000}$ is the fraction (check it is less than 1)

Stage 3: Simplify the fraction if possible

$$\frac{625}{1000}=\frac{125}{200}=\frac{25}{40}=\frac{5}{8}$$

(fractions with denominator 8 are multiples of 0.125)

Converting fractions to decimals

Example: Convert $\frac{3}{8}$ to a decimal.

The fraction bar is a divide sign, so just divide the top by the bottom, either with a calculator or by paper and pencil methods

3 ÷ 8 = 0.375

Challenge: Google “fractions and decimals” and work through the Maths Is Fun page, and try the questions at the end.

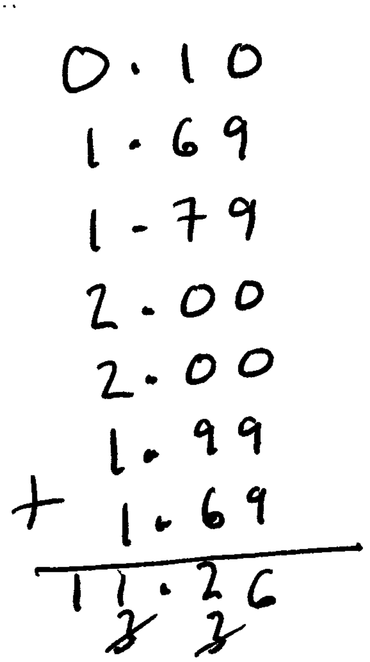
Use a calculator for the divisions at this stage.

Adding and subtracting decimals

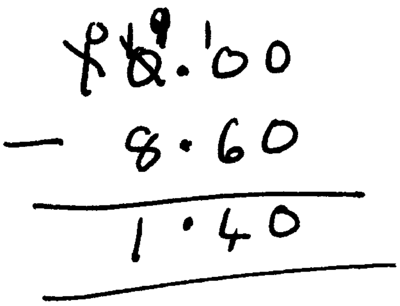
Adding and subtracting decimal numbers works in exactly the same way as adding and subtracting whole numbers in columns.

The scans below show my working out to check my till receipt earlier in this section.

Example: adding items on a till receipt



Example: subtracting £8.60 from £10.00 to check change



Challenge: find a till receipt and check the calculation and change.

Dividing and multiplying by powers of 10

This helps with working out percentages without a calculator (see later).

Multiplying by 10, 100, 1000

Example: 39.27 × 10

Putting the number in place notation columns as below

Th	H	T	U	●	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$	$\frac{1}{100000}$
		3	9	●	2	7			

The digit 3 is worth 30, so multiplying by 10 makes that 300.

The digit 9 is worth 9 units, so multiplying by 10 makes that 90

The digit 2 is worth 0.2 or $\frac{2}{10}$ and multiplying that by 10 gives 2

The digit 7 is worth 0.07 (7p on your till receipt) and multiplying that by 10 makes 0.7

Putting these revised values in the columns gives

Th	H	T	U	●	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$	$\frac{1}{100000}$
	3	9	2	●	7				

As you can see, the numbers have moved to the *left* by one column.

If we write 39.27 × 10 = 392.7

You can see that the decimal point has moved to the right one column.

Either way, there are more digits before the decimal point when you multiply.

Example: 1.47 × 1000 = 1470

Multiplying by 1000 means moving the numbers by 3 columns. We have to fill in the Units column with a zero so the 1 stays 1000 and the 4 stays 400 and the 7 stays 70 in the answer.

Dividing by 10, 100, 1000

Just move the digits (or decimal point) so there are less figures in front of the decimal point, and more behind it.

Example: 19.73 ÷ 100 = 0.1973

Example: 245.3 ÷ 10 = 24.53

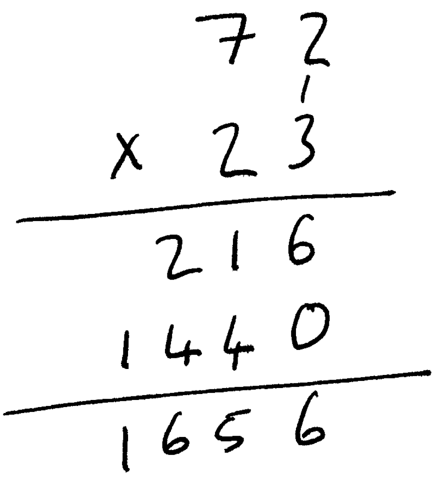
Example: 28.6 ÷ 1000 = 0.0286 here the digits move three columns to the right (or the decimal point moves three places to the left) and so there is an empty space after the decimal point. You have to fill that empty column in with a zero.

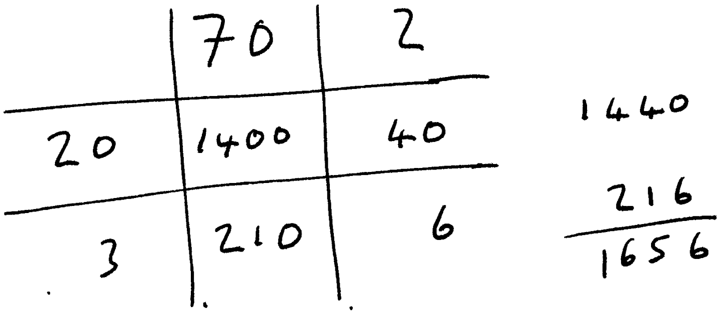
Multiplying decimals without a calculator

Example 2.3 × 0.72

Stage 1: count the total number of decimal places in the two numbers you are multiplying. In this case, there are 3 decimal places (2.3 has one dp and 0.72 has 2 dp)

Stage 2: forget the decimal points and multiply 23 × 72 using any paper and pencil method you like to use. I've scanned two methods in below...

<table>
<tr><th>'Traditional' method</th><th>Box method</th></tr>
<tr><td>


</td><td>


</td></tr>
</table>

So you know that 23 × 72 = 1656. The decimal point is at the end of this whole number.

Stage 3: count back three decimal places, so there are three digits behind and one in front of the decimal point

So 2.3 × 0.72 = 1.656

Stage 4: reality check: 2.3 is about 2 and 0.72 rounds up to 1 to one significant figure. A rough estimate of the answer is 2 × 1 = 2. That is about right for the size of the answer.

Why does this work? When we just ignored the decimal points in the 2.3 and the 0.72, we effectively multiplied the 2.3 by 10 and the 0.72 by 100. So the answer was 10 × 100 = 1000 times too large. The counting back three places in stage 3 is dividing this too large number by 1000.

Challenge: Google “multiplying decimals” and try watching the Khan Academy videos. Find an exercise in a GCSE textbook with two digit by two digit multiplications.

Dividing whole numbers with the answer as a decimal

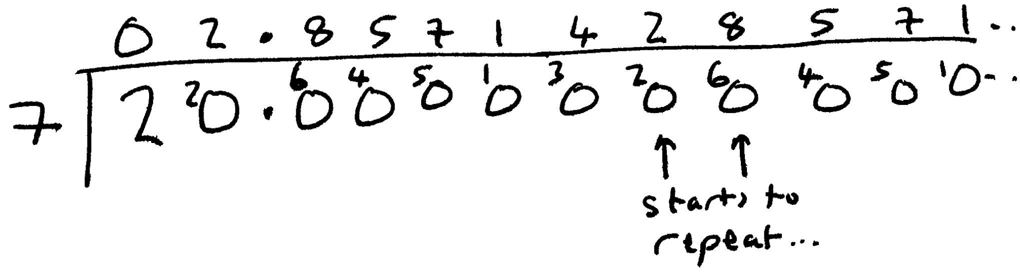
At school you may have said something like “20 ÷ 7 = 2 remainder 6”.

It is quite straight forward to calculate a division with a decimal answer.

Example: 20 ÷ 7

Below is a scan of my 'short division' method for the division calculation.

Because 7 is a prime number and not a factor of 10, this division will repeat the cycle of 6 digits for ever.



Check that you understand how I am carrying out this short division, and where the carry numbers come from at each stage.

Dividing by a decimal

Example: 14.5 ÷ 0.05

Stage 1: you can't actually divide directly by a number with decimal points in it.

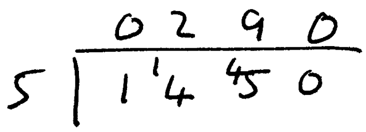
You multiply both numbers by a power of 10 large enough to make the number you are dividing by a whole number.

In this case, we multiply both numbers by 100.

Our new division is 1450 ÷ 5

The answer to this division is the same as the answer to 14.5 ÷ 0.05

Stage 2: Now just divide the numbers as usual, my scan below



Stage 3: There is no stage 3. The answer is 14.5 ÷ 0.05 = 290

When you divide by a decimal number smaller than 1 the answer gets larger!

Challenge: if 1450 ÷ 5 = 290, can you predict the answer to the following calculations without doing any actual calculation?

a) 1.45 ÷ 0.5 b) 145 ÷ 50 c) 0.145 ÷ 500

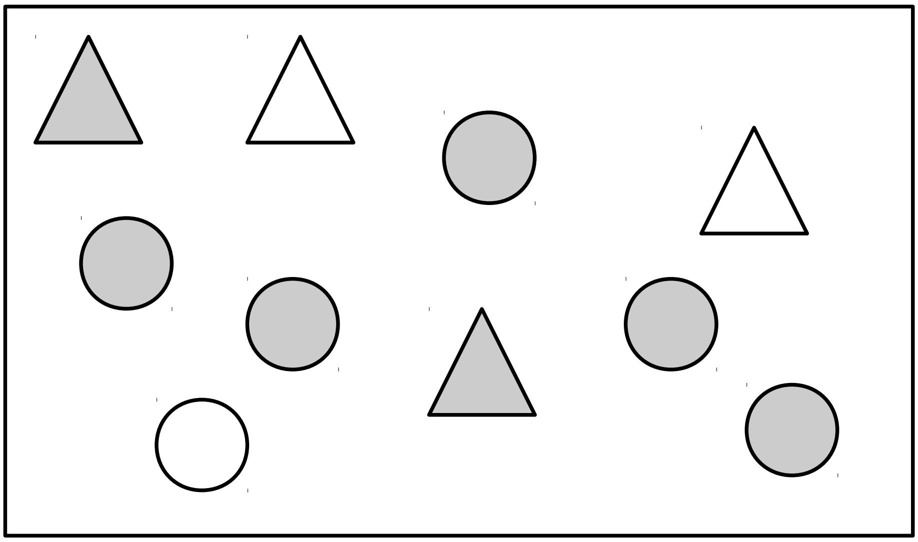
Hint: it is all about powers of 10.

Everyday maths

GCSE Maths has questions about prices, percentage change, profit and loss, loans and interest, and getting the best value for your money! There are also questions about gas and electricity bills, mobile phone tariffs, costing formulas and rounding decimal numbers off to sensible levels of accuracy.

Writing one quantity as a fraction of another

A fraction has a part on top and a whole on the bottom. Look at the shapes in the box



Can you see how I got the fractions below?

Fraction of all the shapes that are circles $\frac{6}{10}=\frac{3}{5}$

Fraction of all the shapes that are triangles $\frac{4}{10}=\frac{2}{5}$

Fraction of all the shapes that are shaded in $\frac{7}{10}$

Fraction of all the shapes that are not shaded in $\frac{3}{10}$

Fraction of the shaded shapes that are triangles $\frac{2}{7}$

Fraction of the shapes that are not shaded that are circles $\frac{1}{3}$

See how what you take as the whole can change sometimes depending on the wording used?

See **Two Way Tables** in the Data section. Challenge: make a two way table showing how the shapes are broken down by shape and by shading.

Examples of fractions from everyday situations

A fraction is a part over a whole: $\frac{part}{whole}$

In most of the situations presented in the exam, you have to decide what the 'part' is and what the 'whole' is.

Depending on the situation described in the question that can involve some adding up or subtracting first.

In many cases you will need to simplify the fraction once you have worked out what the part and whole are.

Example 1: 48 passengers on a bus. 16 are children. What fraction are children?

Fraction that are children is $\frac{16}{48}$ which simplifies to $\frac{1}{3}$

Example 2: The train is late 6 times, on time 14 times and early 10 times in a month. What fraction of train rides are not late?

The part: “Not late” means early or on time, so 14 + 10 = 24 times.

The whole: Total number of train rides is 24 + 6 = 30.

Fraction not late is $\frac{24}{30}$ which simplifies to $\frac{4}{5}$.

In this example you had to add up to find the whole and you had to interpret the question to find out what the part was.

Example 3: There are 18 people on the minibus. 8 are women. What fraction are men?

18 – 8 = 10 men which is the part, and 18 is the whole, so the fraction is $\frac{10}{18}=\frac{5}{9}$

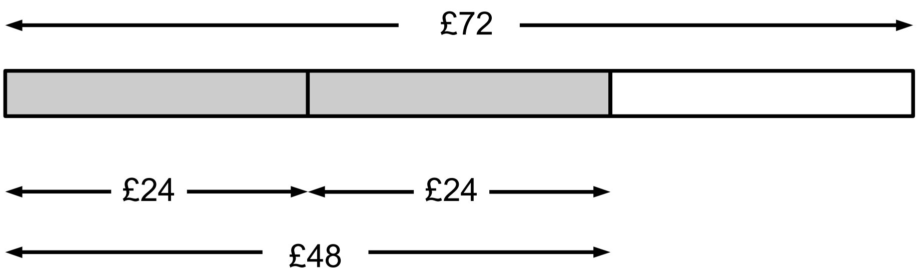
Finding the value of a fraction of a quantity

Example: Find $\frac{2}{3}$ of £72

Stage 1: Divide by the bottom: 72 ÷ 3 = 24

Stage 2: Multiply by the top: 24 × 2 = 48

A Visual approach



Percentages as fractions over 100

“Per cent” means “out of 100” in Latin

So 30% is a way of saying $\frac{30}{100}$

Finding the value of a percentage

Example: find 15% of £480

This can crop up on the calculator or the non-calculator paper as part of a larger question. I'm presenting two non-calculator methods and a calculator method.

Non-calculator: fraction multiplication

Example: Find 15% of £390

Just treat the percentage as if it was a fraction (which it is!).

The word 'of' means multiply in fractions questions...

$$\frac{15}{100}\times\frac{390}{1}=\frac{15}{10}\times\frac{39}{1}=\frac{3}{2}\times\frac{39}{1}=\frac{117}{2}=58.50$$

So 15% of £390 is £58.50

As you can see, you need to be confident at cross cancelling to use this method.

This method is also a lot of work if you have 'difficult' numbers like £187.49 as they are unlikely to cancel.

Non-calculator: dividing by 10, 100 and combining

You can use the place notation system and division by 10 and 100 to find the value of percentages. This non-calculator is better for 'difficult' numbers.

To find the value of 10%, you just divide the amount by 10

To find the value of 1% of an amount, you just divide by 100

Example: find 23% of £126.50

Stage 1: break 23% into 10% + 10% + 1% + 1% + 1%

Stage 2: Find the value of each percentage...
10% is £126.50 ÷ 10 = £12.65

1% is £126.50 ÷ 100 = £1.265   (keep the third place for now!)

Stage 3: Add up two lots of 10% and three lots of 1%

£12.65 + £12.65 + £1.265 + £1.265 + £1.265 = £29.095 ≈ £29.10

You might come across the old VAT rate of 17.5% in some textbooks or older revision material on the Web.

Example: Find 17.5% of £9 500

Stage 1: break 17.5% into 10% + 5% + 2.5%

Stage 2: work out 10% of £9 500 by dividing by 10
£9 500 ÷ 10 = £950

Stage 3: Find the value of 5% by halving the value of 10%

£950 ÷ 2 = £475
Stage 4: Find the value of 2.5% by halving the value of 5%
£450 ÷ 2 = £237.50
Stage 5: add all these together £950 + £475 + £237.50 = £1662.50

Calculator method using formula

The formula is $value = \frac{amount}{100} \times percentage$

Example: Find 19% of £145.75

Just do $\frac{145.75}{100} \times 19 = 27.6925$ and then round the answer to £27.69

The $\frac{145.75}{100}$ part of the formula tells you the value of 1% of the whole.

The $\times 19$ part of the formula multiplies the value of 1% by the number of percent you want.

This formula is closely associated with the 'unitary method' (see later)

Writing one quantity as a percentage of another

Percentages are often used to compare different organisations.

For example, colleges publish percentage pass rates. You can compare a very small college with a huge college by looking at the pass rates as a percentage.

Calculator method using formula

The formula to write *part* as a percentage of the *whole* is $Percentage = \frac{part}{whole} \times 100$

Example: Freda scores 27 out of 40 on her functional skills maths test. What was her mark as a percentage?

$\frac{27}{40} \times 100 = 67.5\%$

The $\frac{27}{40}$ part gives Freda's score as a decimal fraction.

The $\times 100$ part turns the fraction into a percentage.

You can use the same formula to calculate the angles in a pie chart (see later) by using $\times 360$ instead of $\times 100$.

Non-calculator method using fraction multiplication

Example: Write 450 as a percentage of 1500

$$\frac{450}{1500} \times \frac{100}{1}$$

Just the same as the calculator formula except you put 100 over 1 to make it look like a fraction...

$$\frac{450}{1500} \times \frac{100}{1} = \frac{450}{15} \times \frac{1}{1} = 30\%$$

...'Cancelling down' always helps save work with the fraction method calculations.

Calculate the value of a percentage change

Percentages are used when you are talking about pay rises and about discounts.

You have to be able to calculate the result of a change.

The change might be an increase or a decrease.

You have to spot the key words in the question that tell you if it is an increase or a decrease. Then you use the value of a percentage formula to work out the amount of the change, then add the amount on or take the amount off.

Change words

Increase: rise, tax, levy, surcharge,

Decrease: reduction, discount, special offer

Increase example

Example: Raj earns £12 500 and has a 3% pay rise. Calculate his pay after the rise.

Step 1: work out the value of 3% of 12500 (12500 ÷ 100 × 3 = £375)

Step 2: Increase, so add the amount of the rise onto the original amount

£12500 + £375 = £12 875

Decrease example

Example: “Special offer this weekend 30% off all prices”.

The armchair normally costs £299. How much will it cost in the sale?

Step 1: work out the value of 30% of £299 (£299 ÷ 100 × 30 = £89.70)

Step 2: Decrease so subtract £299.00 – £89.70 = £209.30

Use multipliers

You can add or subtract the percentage before finding the value.

Using the same two examples above

Increase wages: new wages 100 + 3 = 103% of old wages, 12500 ÷ 100 × 103 = £12 875

Armchair sale: sale cost = 100 – 30 = 70% of normal cost, 299 ÷ 100 × 70 = £209.30

Express an increase or decrease as a percentage

Percentage increase

Example: Suppose postage stamps go up from 25p to 32p

The increase in price is 32 – 25 = 7p

As a percentage of the <u>original</u> price that is $\frac{7}{25} \times 100 = 28\%$

You can write a formula for the percentage increase

$$\text{percentage increase} = \frac{(New\ price - Original\ price)}{Original\ price} \times 100$$

You always express the change as a percentage of the original price, never the new price.

Percentage decrease

Example: suppose the laptop was reduced from £399 to £299 in a sale.

What was the percentage reduction in cost?

The decrease in price is 399 – 299 = £100

As a percentage of the original cost of the laptop, that is

$\frac{100}{399} \times 100 = 25.062\%$ or about 25%

Mega challenge: Google "GCSE percentage questions" and work through the BBC Bitesize Percentage page including the 'test bite'.

Profit and loss

A small business sells goods or services to create sales income.

The business has to pay out various costs to be able to make those sales.

The difference between the sales income for (say) a month, and the total of all the costs in that month gives the profit for the month.

As a word formula: Profit = Sales – Costs

GCSE Maths exams have a variety of problems based on this formula, and some of those may involve percentage calculations.

Example: suppose Amjad buys a car for £8 500 and sells it a year later for £5 000. Calculate his percentage loss on the car

The loss on the car was 8 500 – 5 000 = £3 500

As a percentage of the original cost of the car, the loss was

$$\text{Percentage loss} = \frac{(buying\ price - selling\ price)}{buying\ price} \times 100$$

Putting numbers in $\frac{(8\,500 - 5\,000)}{8\,500} \times 100 = 41.176\%$ or about 41% loss.

Example: Flossie buys a box of mangoes for £16. The box holds 48 mangoes. She wants to make 60% profit on the mangoes to be able to pay for the market stall and to make some money. What price should she set for a mango?

Stage 1: Add 60% profit onto the cost of the mangoes

$\frac{16}{100}\times 60=£9.60$ is the amount of profit needed, so Flossie has to achieve sales of £16 + £9.60 = £25.60

Stage 2: Divide the result by 48 to find the price of one mango

£25.60 ÷ 48 = £0.53333... ≈ 0.54

Flossie rounds up to be sure of making her profit

Discounts and special offers comparisons

Some shops give percentage discounts when they have a sale

Some shops make '2 for the price of 3' offers (I have even seen '5 for the price of 6' in a well known supermarket)

Some shops use fractions to convey their reductions

A popular GCSE question (also found in Functional Skills) is to compare several special offers and say which is the best.

Example: Two clothes shops are each having a sale. Rhiannon spots a jacket she likes in each shop. Below are the special offers...

Shop A: “One third off all marked prices this week”, jacket has marked price of £19.99

Shop B: “25% of everything this week”, jacket was sold at £15.99 before the sale

Compare the prices. Which jacket is cheaper?

Stage 1: Calculate the cost of the jacket from Shop A

£19.99 ÷ 3 = £6.66 (rounding off). As the offer says “One third off” we subtract the £6.66

£19.99 – £6.66 = £13.33 sale price

Stage 2: Calculate the cost of the jacket from Shop B

The value of the discount: $\frac{15.99}{100}\times 25=3.9975$ Rounding up to £4.00

Subtract the discount from the original price £15.99 - £4 = £11.99

Stage 3: compare the prices: Shop B is cheaper by £1.34

Challenge: find a GCSE textbook and look for 'star' questions or 'A03' questions. These are the problem solving questions which are worth 4 or 5 marks and which often involve comparisons such as this example.

Proportion and Unitary Method

Example: 12 apples cost £2.40. What is the cost of 7 apples?

Stage 1: find cost of one apple by dividing £2.40 ÷ 12 = £0.20

Stage 2: multiply the cost of one apple by the number of apples you want to buy

£0.20 × 7 = £1.40

You can say that the cost is **proportional** to the number of apples you buy

One thing is proportional to another if when you double the first thing, the second thing doubles.

Other examples of **direct proportion** include

Weight on a spring and the extra length of the spring

Area of a wall and the amount of paint you need to paint the wall

Examples of relationships that might **not** be proportional include

Number of minutes of mobile phone calls and monthly fee (many contracts give 'free' minutes so the direct proportionality is lost)

Cost of paint and number of litres of paint (larger tins usually cost less per litre)

The example above uses the 'unitary' method because in stage 1 you worked out the cost of one apple. The cost of a single item is often called the **unit price**.

This section also links with **finding the value of a fraction** above and **conversion graphs** below.

Best value

Next time you are in the supermarket, look at the price tags on the shelves for coffee, or for rice or for olive oil.

You will see the price of the jar/packet/bottle then underneath in small type you will see the price per 100g or 100ml. This second small price is always for the same amount of product and the supermarket provides it so that you can directly compare prices, even for different sized containers.

Example: Happy Trader Instant Coffee comes in eco-packs that contain 350g and each pack costs £5.75. Calculate the cost per 100g.

Stage 1: 350g is 350 ÷ 100 = 3.5 lots of 100g

Stage 2: Calculate cost per 100g by dividing £5.75 ÷ 3.5 = 1.642857143 ≈ £1.64 per 100g

Example: Floor Sweeper Instant Coffee is sold in enormous 800g tins for £14.99 per tin. Calculate the cost per 100g

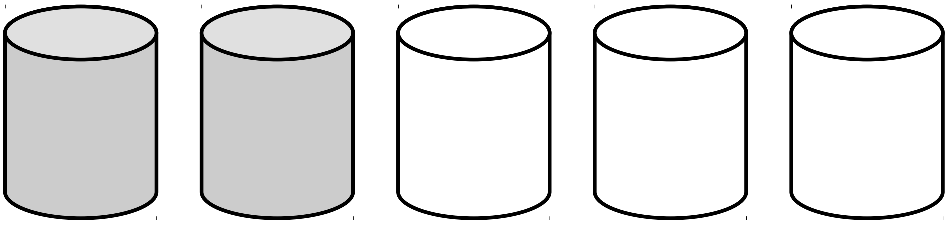
Stage 1: 800 ÷ 100 = 8 lots of 100g

Stage 2: £14.99 ÷ 8 = 1.87375 ≈ £1.87 per 100g

Comparing the costs, the Happy Trader Instant Coffee is actually cheaper per 100g.

NB: Brands are entirely fictional!

Ratios



Suppose you have 2 tins of grey paint and 3 tins of white paint and you mix them all together to get a light grey colour.

You have mixed the paints in the ratio 2:3 of grey to white.

The colon separates the two ratio numbers.

The fraction of the mixture that is grey paint is $\frac{2}{5}$ and the fraction that is white is $\frac{3}{5}$.

There are 5 equal parts all together when you mix the paint, so the fractions have denominator 5.

Example: the instructions on the label of the bottle say “mix one part SuperGrow with 5 parts water”. Write this as a ratio

The ratio of SuperGrow to water is 1:5

Example: “Mix 50g of power A with 1.5Kg of power B”. Write these instructions as a ratio in its simplest form

Stage 1: Convert to same units

Convert 1.5Kg to 1500g

Stage 2: Form the ratio and simplify

Mix powder A with powder B in the ratio 50:1500 = 1:30

Notice that you have to change the units so that both parts are measured in the same units before you can form the ratio.

Then you can divide both of the numbers in the ratio by their lowest common multiple

Dividing a quantity in a ratio

Example: Grey and white paint is mixed in the ratio 2:3.

Kamaljit needs 20 litres of the mixture. How much grey paint and how much white paint does he need?

Stage 1: Find the value of one equal part

2 + 3 = 5 equal parts and 20 litres in total. So one equal part must be 20 ÷ 5 = 4 litres

Stage 2: Multiply the ratio number by the number of equal parts

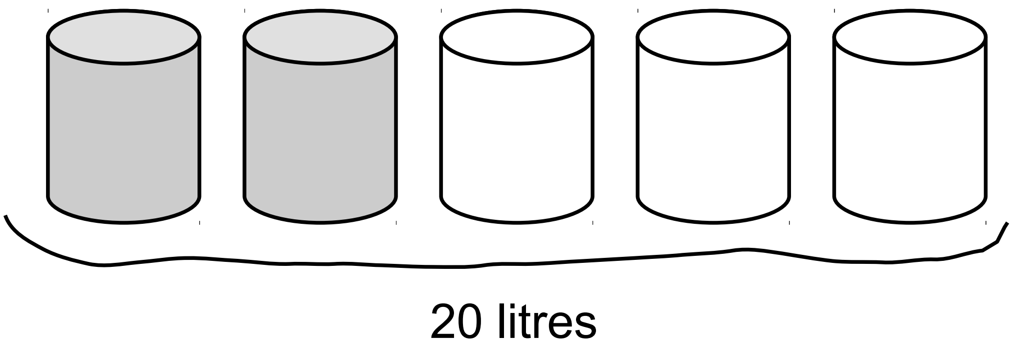
Grey paint 2 × 4 = 8 litres

White paint 3 × 4 =12 litres

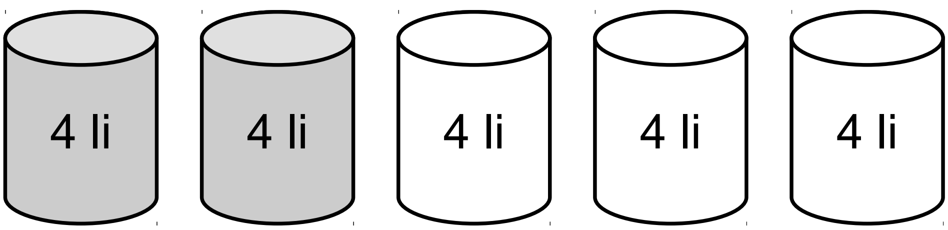
Stage 3: check by adding

8 + 12 = 20 litres so correct

Visual presentation...

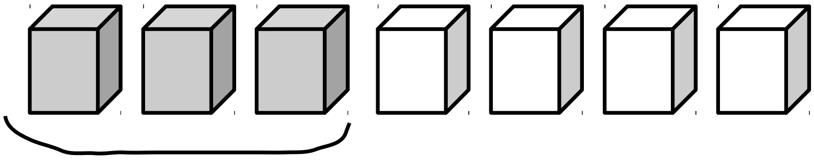


Each tin same size, so



Add up the grey tins 4 + 4 = 8 litres of grey
Add up the white tins 4 + 4 + 4 = 12 litres of white

Finding the value of one part given the value of another



7.5kg

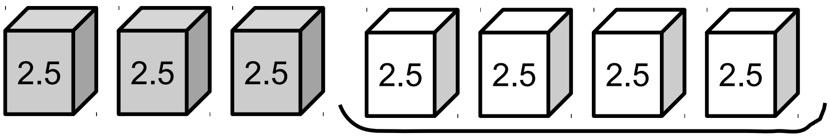
Example: suppose you mix gravel and sand in the ratio 3:4. You have 7.5Kg of gravel. How much sand do you need to mix with the gravel to get the right quantity?

Stage 1: You know that 3 parts of gravel weighs 7.5Kg

Stage 2: One equal part is 7.5 ÷ 3 = 2.5kg

Stage 3: You need 4 parts sand so 4 × 2.5 = 10Kg of sand

Visual version of stage 2 and 3



10 kg

Ratios with fractions

Ratios with fractions or decimals can be simplified by multiplying by a common denominator (not always the *lowest* common denominator) and then simplifying in the usual way.

Below are examples for decimal fractions in ratios and for proper fractions in ratios

Example: Simplify the ratio 1.25 : 0.8

Stage 1: multiply both numbers by 100 (as one of the numbers in the ratio has 2 decimal places) 1.25 : 0.8 equivalent to 125 : 80

Stage 2: simplify 125 : 80 = 25 : 16

Example: Simplify the ratio $\frac{3}{4}:\frac{5}{8}$

Stage 1: Lowest common denominator is 8, so multiply both fractions by 8

$$\frac{3}{4}\times 8:\frac{5}{8}\times 8=6:5$$

Stage 2: No need to simplify because we multiplied by *lowest* common denominator.

Ratio challenge: Google “GCSE Ratio questions” and work through the BBC Bitesize page.