**Getting from $ax^2 + bx + c = 0$ to the Quadratic Formula**

The formula for the general quadratic equation is

$$ax^2 + bx + c = 0$$

You can 'complete the square' on this general equation to derive the well known quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Completing the square**

The first step is to divide through $ax^2 + bx + c$ by $a$ so the coefficent of the first term is 1

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Then you can 'complete the square' by taking half the coefficient of $x$ in the usual way

$$\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0$$

Simplifying $\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$

$$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

Now taking the constants outside the square to the other side

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

The last step in the completing the square solution method has you taking the square root on both sides

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2}{4a^2} - \frac{c}{a}}$$

But that does not look very much like the 'Quadratic Formula' as given on the official formula sheet!

**Tidying up**

You need to use a few algebraic rules to transform the formula into its usual form.

The first thing to do is take $\frac{b}{2a}$ to the other side so you have $x$ on the left and the right hand side consists solely of constants

$$x = -\frac{b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{c}{a}}$$

Then you have to puggle with the fractions *under* the square root a bit.

What we want to do is to write $\frac{c}{a}$ over the denominator $4a^2$, so just multiply by 1 in disguise $\frac{c}{a} \times \frac{4a}{4a} = \frac{4ac}{4a^2}$ which gives

$$x = -\frac{b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{4ac}{4a^2}}$$

Then write the difference inside the square root over the denominator

$$x = -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

Then because $4a^2$ is a perfect square, you can take it out of the square root as $2a$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

And finally you can write the whole right hand side over the common denominator of $2a$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So you've got to the Quadratic Formula as written on the official formula sheet.

**Bonus: Equation of the axis of symmetry of the quadratic graph**

While we are doing algebra with the quadratic formula, just unpack the formula to give each of the two possible solutions $x_1$ and $x_2$

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

and

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Remember that a parabola, the shape of the quadratic graph, has a vertical mirror line, and this line must be half way between the two roots or solutions of the quadratic equation.

To find half way between, say $P$ and $Q$ you just find $\frac{P+Q}{2}$.

Adding up the two solutions $x_1 + x_2$ gives

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b + \sqrt{b^2 - 4ac} + -b - \sqrt{b^2 - 4ac}}{2a}$$

The two $\sqrt{b^2 - 4ac}$ terms of opposite sign neatly cancel out to zero so you have

$$x_1 + x_2 = \frac{-b + -b}{2a} = \frac{-2b}{2a}$$

So finally

$$\frac{x_1 + x_2}{2} = \frac{-b}{2a}$$

The equation of the line of symmetry for a quadratic graph is $x = -\frac{b}{2a}$ which is a vertical line.