Maths Foundation Topic Guide 1

This topic guide lists the basic facts about the topics and skills we will cover in the first six weeks of the GCSE Maths course on the new grade 9 to grade 1 syllabus.

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Useful Web sites

- Corbett Maths https://corbettmaths.com/
- Maths is Fun https://www.mathsisfun.com/
- Hegartymaths on YouTube
  https://www.youtube.com/user/HEGARTYMATHS
  Just go to youtube and search for HEGARTYMATHS and subscribe to Mr Hegarty's channel
- BBC Skillswise www.bbc.co.uk/skillswise
  Handy for brushing up your English and everyday maths skills

Equipment

You must have...

- Pens, pencils, ruler, eraser, paper and a folder
- A scientific calculator (I will have examples available in the first lesson)
- A set of maths instruments (demonstration in the first lesson)

Other things that might be useful

- A small address book for recording the definitions of 'maths words'
- Index cards for making your own flash card memory tests and for revision notes

Some symbols you need to know

Symbol	Meaning	Example
<	Less than	“7 < 10” is true
>	Greater than	“–3 > –5” is true
≤	Less than or equal to	“3 ≤ 8” and “5 ≤ 5” are both true
≥	Greater than or equal to	“7 ≥ 6” and “10 ≥ 10” both true
x^2	Square of a number	“Square of 3 = 3 × 3 = 9”
x^3	Cube of a number	“Cube of 4 = 4 × 4 × 4 = 64”
$\sqrt{16}$	Square root	$\sqrt{16}=4$ And $\sqrt{100}=10$
$\sqrt[3]{125}$	Cube root	$\sqrt[3]{125}=5$ And $\sqrt[3]{1000}=10$
+	Add, sum, total	“The sum of their ages was 38”
–	Subtract, difference	“The difference in their heights was 10cm”
×	Multiply, product	“The product of three numbers is 60”
÷	Divide	“How many 50g servings can you get from a 500g bag of rice?”

BIDMAS

BIDMAS stands for **B**rackets **I**ndex **D**ivide **M**ultiply **A**dd **S**ubtract

This phrase tells you the order of operations when you are calculating the value of something like 4 + 3 × 5.

By convention, you have to complete the multiplication before adding, so

4 + 3 × 5 = 4 + 15 = 19

A trickier one is 20 – 30 ÷ 5. The division has to be completed before the subtracting, so

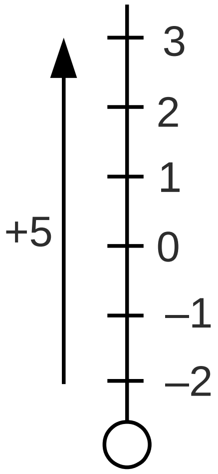
20 – 30 ÷ 5 = 20 – 6 =14

Index is another word for power so 3^4 means 3 × 3 × 3 × 3 = 81 and you read 3^4 as "three to the power 4". An example would be 3×5^2. You work out five to the power 2 first, then multiply by 3.

$3 \times 5^2 = 3 \times 25 = 75$.

Brackets can be used to change the sequence, so (4 + 3) × 5 = 7 × 5 = 35.

Negative numbers



Negative numbers have values less than zero.

When you have an overdraft, you have less than zero money.

Your fridge stores food at about –4 °C and the freezer can reach –20 °C.

The thermometer on the left shows warming up from –2 °C to +3 °C. The difference between the two temperatures is 5 degrees.

So 3 – –2 = 5. (“two minuses make a plus”)

Adding and subtracting negative numbers

Adding / subtracting	Both same sign Add and give sign	–3 – 5 = –8 –7 – 12 = –19 3 + 8 = 11
	Opposite signs Find difference and give sign of largest	–3 + 5 = 2 –7 + 12 = 5 9 + – 6 = 3 –12 + 5 = –7

Multiplying and dividing negative numbers

Dividing / multiplying	Same signs Answer positive	5 × 4 = 20 –12 × –3 = 36
	Opposite signs Answer negative	–6 × 4 = –24 –50 ÷ 5 = –10

When you are adding/subtracting, the sign of the answer will depend on the size of the negatives and the size of the positives.

When you are multiplying and dividing, you can predict the sign before you start.

These rules may seem strange at first but they become less strange with practice.

Types of number

Even and odd numbers

Even numbers are 0, 2, 4, 6, 8, 10, 12, 14, 16... You can always divide an even number into two equal amounts

Odd numbers are 1, 3, 5, 7, 9, 11, 13, 15... You can't split an odd number into two even piles, there will always be one left over

You can tell if a large number is odd or even just by looking at the last digit. So 319 756 is even because 6 is even and 1 673 is odd because 3 is odd.

Odd + odd = even (the two ones left over make a 2)

Odd + even = odd (there is still one left over when you've added them)

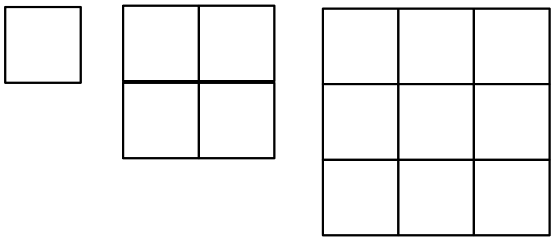
Even + even = even

Square Numbers

Square numbers are the numbers 1, 4, 9, 16, 25, 36, 49, 64, 81, 100...

You make a square number by picking any whole number and multiplying it by itself, so 12 × 12 = 144 so 144 is a square number. You can use the power or index notation for squaring $12^2 = 144$.

The name 'square' number comes from multiplying the lengths of the sides of a square to find its area. Try drawing diagrams to show the first 5 square numbers on squared paper – it helps. The first three are drawn below...

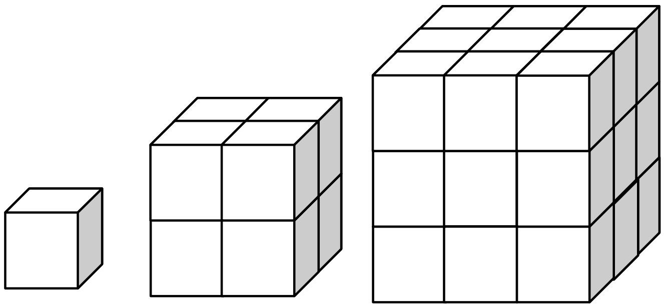


Cube Numbers

Cube numbers are 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000...

You make a cube number by picking any whole number and multiplying it by itself and by itself again, so 11 × 11 × 11 = 1 331. Using index notation you get $11^3 = 1\ 331$.

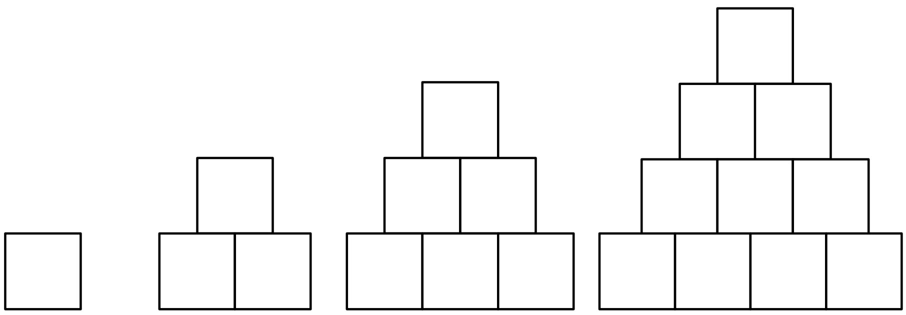
The name 'cube' for power three comes from multiplying the lengths of the sides of a cube to find the volume. You can build cube numbers from dice. The diagram below shows 1^3, 2^3 and 3^3.



Triangle Numbers

Triangle numbers: 1, 3, 6, 10, 15, 21, 28...

These numbers are best explained by a visual pattern...



Prime Numbers

Prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31... Prime numbers have no factors except themselves and 1. See the *Factors and Multiples* section.

Whole numbers

Whole numbers: numbers that don't have any fractional parts. 1, 3, 2017 are whole numbers but 6.7 and 4¾ are not whole numbers

Integers

Integers: whole numbers including zero and negative numbers.

So –45, 0 and –6 are integers as well as 25 and 625

Numbers like –3.5 and –4½ are not integers

Fractions (see later for much more)

Fractions: 0.67, $\frac{2}{3}$, 42% are examples of fractions. You can make a whole number look like a fraction by putting it over a denominator of 1, so $4=\frac{4}{1}$ will work like a fraction when you are doing fraction arithmetic.

Mixed numbers

Mixed numbers: 3½, 12⅝ are examples of mixed numbers.

Factors, Multiples and Prime Factors

Factors

Factors: 3 is a factor of 12 because when you divide 12 by 3 there is no remainder.

Question: List all the factors of 48.

Start with 1 × 48, then 2 × 24, then 3 × 16, then 4 × 12, then 6 × 8.

Factors come in pairs.

Multiples

Multiples: The multiples of 8 are 8, 16, 24, 32, 40...

The multiples of a number are just the times table of that number. You can generate the multiples by adding on another 8 to the last total

Highest Common Factor

Highest common factor (HCF) of two or more numbers: The common factors of 12 and 18 are 1, 2, 3 and 6. So we call 6 the HCF of 12 and 18.

Lowest Common Multiple

Lowest common multiple (LCM) of two or more numbers: The LCM of 8 and 12 is 24. Both 8 and 12 go into 24 without a remainder, and 24 is the smallest number where that is true.

Prime numbers

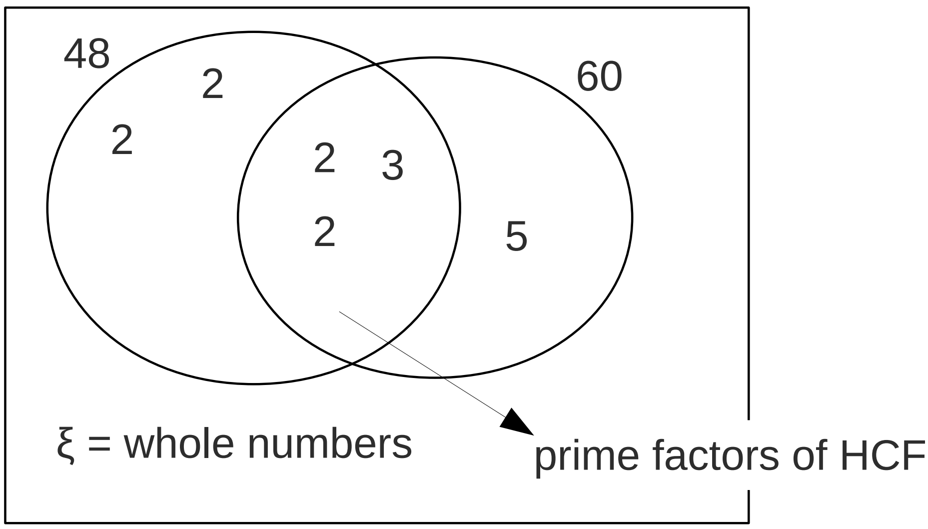
Prime numbers: Some numbers have exactly two factors, the number itself and 1. Those are called prime numbers, and the first few are 2, 3, 5, 7, 11, 13, 17, 19. In the 17th century, they counted 1 as a prime number but not any more. Whole numbers that are larger than 1 and not prime numbers are called composite numbers.

Prime factors

Prime factors: You can write any composite number as a product of a certain set of prime numbers. For example, $12 = 2 \times 2 \times 3 = 2^2 \times 3$, another example is $60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$. See how the first three prime factors of 60 are the prime factors of 12? That means 12 has to be a factor of 60. You can find the prime factors of a composite number using the prime factor tree or repeated division.

Venn diagram and prime factors

The prime factors of 48 are $2 \times 2 \times 2 \times 2 \times 3$ and the prime factors of 60 are $2 \times 2 \times 3 \times 5$. You can put those factors into a **Venn diagram**...



The factors in the **intersection** of the two circles are the prime factors of the

HCF of 48 and 60

The prime factors of the LCM of 48 and 60 are the product of all the prime factors in the two circles, the **union** of the 48 set and the 60 set.

Powers and roots

Powers

Powers: $5^8 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 390\ 625$

You can **multiply** powers of the same number by **adding the powers**, so

$5^3 \times 5^4 = 5^7$

You can **divide** powers of the same number by **subtracting the powers**,

$$\frac{5^{12}}{5^9} = 5^3$$

Question: Write $\frac{2^{10} \times 2^5}{2^{11}}$ as a single power of 2.

Answer: Simplify the top by adding powers $\frac{2^{10} \times 2^5}{2^{11}} = \frac{2^{15}}{2^{11}}$

Square roots

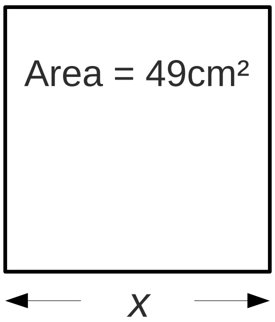
Then simplify the division by subtracting the powers $\frac{2^{15}}{2^{11}}=2^4$

Square root: Find the square root of 10 and round your answer to 2 decimal places: $\sqrt{10}=3.16227766017\approx 3.16$

You can write down the square root of a square number, $\sqrt{36}=6$.

Square roots come up in work about area as well

Question. The square below has area 49 cm².
Write down the value of *x*.



Answer: the length of the side is the square root of the area $\sqrt{49}=7\,\text{cm}$

Cube roots

Cube root: Find the cube root of 70 and round your answer to 2 decimal places. $\sqrt[3]{70}=4.12128529981\approx 4.12$

You can write down the cube root of a cube number, so $\sqrt[3]{125}=5$

Place value, rounding and estimation

Numbers and words

Numbers in words: One hundred and twelve thousand and fifteen is written as 112 015. Google “English number words” if you need a reminder.

Place value

Place value: Each digit in a number has a value according to the column that the number is in. Have a look at the example below…

Millions		Thousands			Hundreds/tens/units				Decimals	
10M	M	100T	10T	T	H	T	U	•	$\frac{1}{10}$	$\frac{1}{100}$
3	2	5	0	6	7	0	4	•	1	8

The place value of the digit 5 in this number is 500 000 or *five hundred thousand*.

The place value of the digit 8 in this number is $\frac{8}{100}$ or *eight hundredths*.

It is very unusual to keep a number to 10 *significant figures* like this, most of the time you round off to the nearest million, or perhaps the nearest hundred thousand.

Rounding

Rounding: To round the number 4 367 to the nearest *hundred*, you look at the digit in the *tens* column – **one place *after* the place you want to round to**.

If the value one place after is 0, 1, 2, 3, 4 leave the hundreds digit

If the value one place after is 5, 6, 7, 8 or 9 round the hundreds digit up

In this case, the tens digit is 6, so we round the hundreds digit up to 4 and then you write zeros in the tens and units so 4 367 = 4 400 to the nearest 100.

The number 5 824 rounded to the nearest <u>hundred</u> is 5 800

More examples: 13 296 = 13 000 to the nearest thousand

13 806 = 14 000 to the <u>nearest</u> thousand

Decimal places

The numbers after the decimal point are called the decimal places.

Example: Round 14.93792 to two decimal places
Keep the 9 and 3 digits, look at the 7 digit, which is 5 or over, so round the 3 digit to a 4, so 14.93792 = 14.94 to two decimal places.

See how we don't put extra zeros after the two places we want to keep.

Example: Round 0.295368 to three decimal places
Keep the 2, 9 and 5 digits.
Look at the 7 digit, which is less than 5, so the 5 digit we kept does not change
So 0.295368 = 0.295 to three decimal places
If we wrote 0.295000 that would be incorrect, no trailing zeros

Significant figures

Significant figures start from the first digit on the left that is not a zero

Rounding works in the same way as for decimal places.

sig fig	1st	2nd	3rd	4th		5th	6th
	4	0	9	6	•	2	5

The first significant digit in the number 4096.25 is 4 in the thousands column, and the second significant digit is zero in the hundreds column.

Rounding 4096.25 to two significant figures gives 4100. The digit 9 in the tens column says we have to increase the 0 to 1. But then you have to put zeros in the tens and units column as you have to have place holders so the 4 stays in the thousands and the 1 stays in the hundreds.

Numbers less than 1: best shown by example

Suppose you do 3 ÷ 700 on your calculator and the display will show something like 0.00428571428571. The table below shows the significant figures…

	•			1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th
0	•	0	0	4	2	8	5	7	1	4	2	8	5	7	1

To round this number to 3 significant figures (3 s.f.) you keep the 4, 2 and 8 and look at the 4th significant figure which is 5, so the 8 rounds up to 9. Putting that all together you end up with 0.00429.

See how the zeros after the decimal point but in front of the 4 are not significant, but they are very important because they keep the 4 in the thousandths column.

Estimating the results of calculations

In estimation, you get a rough value for a calculation.

Example: Estimate the answer to 23.7 × 99.2

Answer: Round both numbers to 1 significant figure, so the calculation becomes 20 × 100, then complete the calculation using BIDMAS so the estimated answer is 2 000.

Eaxmple: Estimate the answer to $\frac{9.95^2}{19.95-14.98}$

Answer: Round the numbers to 1 significant figure $\frac{10^2}{20-15}$

Then complete the calculation using BIDMAS rules $\frac{10^2}{20-15}=\frac{100}{5}=20$

So your estimate is 20 for the answer.

Putting decimals in order of size

Example: Put these numbers in order of size starting with the smallest

7.04, 0.74, 0.07, 0.704, 0.074, 0.7

Answer: Pad out all the numbers so they all have the same number of decimal places as the number with the largest number of decimal places

7.040, 0.740, 0.070, 0.704, 0.700

Now it is easier to see which is bigger and which is smaller by pretending these are whole numbers in the thousands

0.070, 0.700, 0.704, 0.740, 7.040

Now get rid of the extra zeros at the end

0.07, 0.7, 0.704, 0.74, 7.04

Find half way between two decimal values

Example: find the number that is half way between 6.1 and 6.8

Answer: add the numbers up to find the total then divide the total by 2.

6.1 + 6.8 = 12.9 ÷ 2 = 6.45

Standard form

If you read a number like 8 500 000, you have no way of knowing how many of the zeros are significant.

8 500 000 written in standard form is 8.5×10^6. As you can see the significant figures are separated from the power of 10.

The significant figures are always written as a number x so that $1 \le x < 10$

The power of 10 is positive for numbers larger than 1 and negative for numbers smaller than 1.

The number 0.000375 would be written as 3.75×10^{-4} in standard form

Multiplying numbers in standard form

Work out: $2.1 \times 10^6 \times 5.3 \times 10^2$

Multiply the numbers: $2.1 \times 5.3 = 11.13$

Multiply the powers of 10: $10^6 \times 10^2 = 10^8$

Adjust so that the number is between 1 and 10: 1.113×10^9

Dividing numbers in standard form

Dividing numbers in standard form: Follow the same pattern

- Divide the numbers
- Divide the powers of 10
- Adjust so that the number is between 1 and 10

Adding and subtracting numbers in standard form

- Modify so both numbers have same power of 10

- Add/subtract

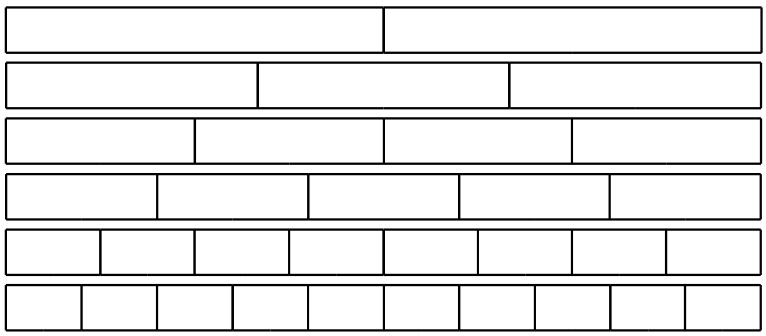
- Return to standard form by adjusting the number and power of 10

Common fractions

Learn this table

Fraction	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{20}$
Decimal	0.5	$0.\overline{3}$	0.25	0.2	0.125	0.1	0.05
Percent	50%	$33.\overline{3}\%$	25%	20%	12.5%	10%	5%

Visual presentation (try a web search for ‘fraction wall’ to find more variations)



Find the value of a fraction

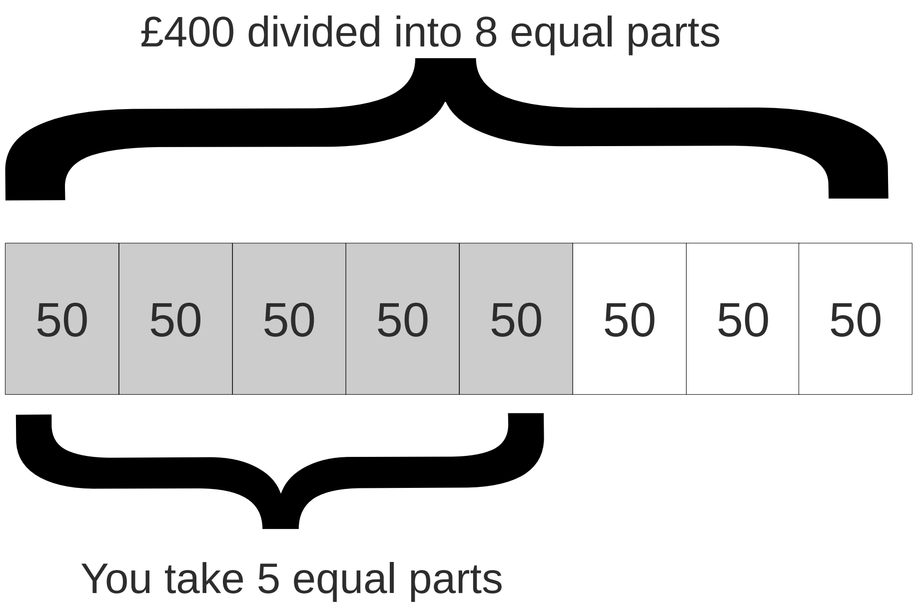
The top of a fraction is called the numerator and the bottom is called the denominator.

Question: Find $\frac{5}{8}$ of £400

Answer: The fraction $\frac{5}{8}$ means that you have divided the whole into 8 equal parts and you have taken 5 of them. So divide the £400 by the bottom, 400 ÷ 8 = £50 per part

Then total up your five parts: 50 × 5 = £250

The visual presentation of the question is sometimes called the Singapore Bar Method...



Another arithmetical method: $\frac{5}{8}$ of £400 means the same as $\frac{5}{8}$ × £400 so look at the multiplying fractions section later in this guide

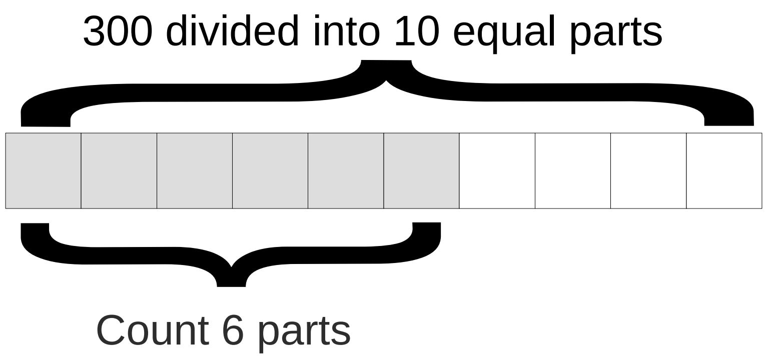
Find the value of a decimal

Question: find 0.6 of 300.

Answer: just multiply so 0.6 × 300 = 180

Alternative method: remember that 0.6 = $\frac{6}{10}$ and divide by 10 and multiply by 6 to give 300 ÷ 10 = 30 × 6 = 180

Visual: You can also use the Singapore Bar Method with 10 bars equal to 300 then take 6 of them.



Find the value of a percentage

Question: work out 65% of £800

Non-calculator method: Split 65% into 50%, 10% and 5%

50% is half, so £400.

10% is one tenth, so 800 ÷ 10 = £80

5% is half the value you worked out for 10% so £40

Adding these up gives 400 + 80 + 40 = £520

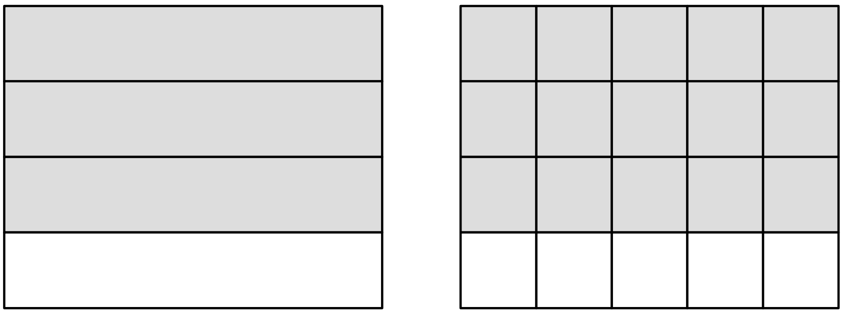
Calculator method: whole ÷ 100 to find 1% then multiply by percentage

800 ÷ 100 = £8 so 1% is worth £8. 65 × 8 = £520

Fraction method: 65% = $\frac{65}{100}$ so just find $\frac{65}{100}$ of 800 (divide by 100 multiply by 65)

Equivalent fractions and simplest form

Look at the two rectangles below… the fraction shaded in each rectangle is clearly the same



$\frac{3}{4}$ $\frac{15}{20}$

This visual image tells us that $\frac{3}{4}=\frac{15}{20}$

You could draw rectangles to show that $\frac{3}{4}=\frac{6}{8}=\frac{9}{12}=\frac{12}{16}=\frac{75}{100}$ and so on

All of these fractions are equivalent to each other.

The fraction $\frac{3}{4}$ is called the simplest form because the numerator and denominator have no common factors other than 1.

Equivalent fraction puzzles

Question: Find the missing denominator in $\frac{2}{5}=\frac{?}{30}$

Answer: You had to multiply the 5 by 6 to get 30, so you have to multiply the 2 by the same number to get 12, so $\frac{2}{5}=\frac{12}{30}$

The best thing to do is to get a worksheet full of these puzzles and work through them all. That will improve your knowledge and speed with the multiplication tables as well as laying a firm foundation for arithmetic with fractions.

Comparing fractions

Question: which is larger, $\frac{1}{3}$ or $\frac{1}{5}$?

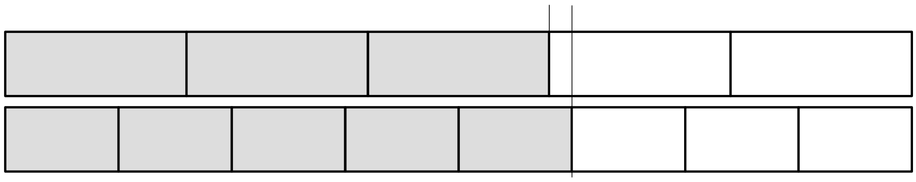
Answer: these are both 'unit fractions', so we can just compare the denominators and say that $\frac{1}{3}$ is largest because you are only dividing an amount into 3 parts, instead of the same amount divided into 5 parts.

Question: which is larger, $\frac{3}{5}$ or $\frac{5}{8}$?

Answer using equivalent fractions: use a common denominator of 40 and write each of the two fractions as a fraction over 40 then compare the numerators.

$\frac{3}{5}=\frac{24}{40}$ and $\frac{5}{8}=\frac{25}{40}$ so $\frac{5}{8}$ is the larger fraction.

Visual answer: using *two* 'Singapore bars' back to back...



You could draw the 'Singapore bars' using squared paper. The small gap will be $\frac{1}{40}$ of the length of the bars.

Putting fractions in order of size

Same idea as comparing fractions – it is best to use a common denominator for all of the fractions.

Question: Put these fractions in order of size starting with the smallest

$$\frac{2}{5}\ ,\ \frac{3}{4}\ ,\ \frac{3}{8}\ ,\ \frac{7}{20}$$

Answer: Use 40 as common denominator

$$\frac{2}{5}=\frac{16}{40}\ ,\ \frac{3}{4}=\frac{30}{40}\ ,\ \frac{3}{8}=\frac{15}{40}\ ,\ \frac{7}{20}=\frac{14}{40}$$

so comparing the numerators, the correct order is $\frac{7}{20}\ ,\ \frac{3}{8}\ ,\ \frac{2}{5}\ ,\ \frac{3}{4}$

Answer using calculator: Divide the numerator by the denominator to find the decimal equivalent,

$$\frac{2}{5}=0.4\ ,\ \frac{3}{4}=0.75\ ,\ \frac{3}{8}=0.375\ ,\ \frac{7}{20}=0.35$$

then use the decimals to decide the correct order

Visual presentation: Find a 'fraction wall' that shows the fraction families

You can see how the fractions sort themselves out, and how $\frac{3}{4}$ is clearly the largest as it is more than half. The other three are quite close together.

Find value of fraction of a large number: A student thought up the idea of a pie chart for these questions. Just work out the number of degrees for each fraction and use that to put them in order. 360 has a *lot* of factors!

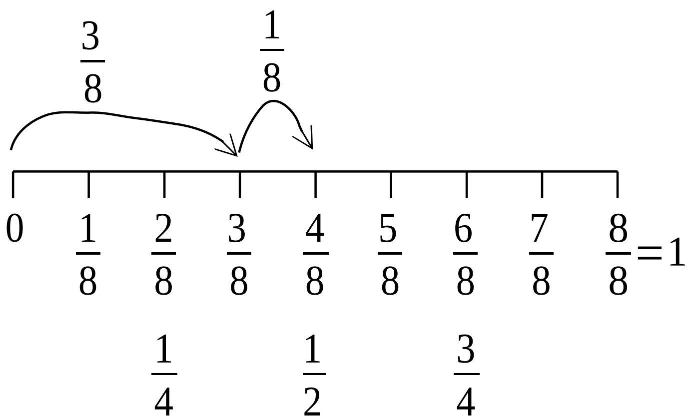
Adding fractions

Question: Work out $\frac{1}{8}+\frac{3}{8}$

Answer: Because the denominators are the same, you can just add the tops.

You are really counting the number of eighths $\frac{1}{8}+\frac{3}{8}=\frac{4}{8}=\frac{1}{2}$

Visually: I've counted out the addition using arrows along a number line with eighths. I've also shown the equivalents on the scale.



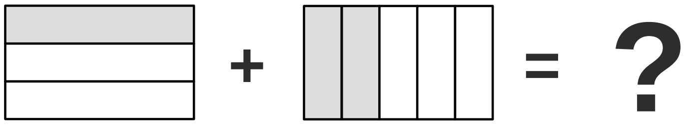
Question: Work out $\frac{1}{3}+\frac{2}{5}$

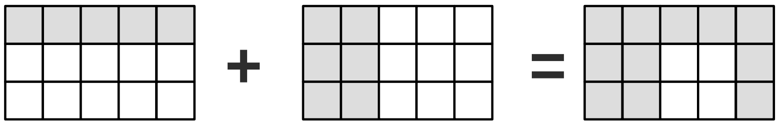
Answer: you have to write each of the fractions over a common denominator in the same way as comparing fractions

$\frac{1}{3}=\frac{5}{15}$ and $\frac{1}{3}=\frac{5}{15}$

Now the denominators are the same so we can add $\frac{5}{15}+\frac{6}{15}=\frac{11}{15}$

Visual presentation: using rectangles of the same area to represent each fraction...





Subtracting fractions

Subtracting works in the same way as adding except that once you have the two fractions written over a common denominator, you subtract the numerators.

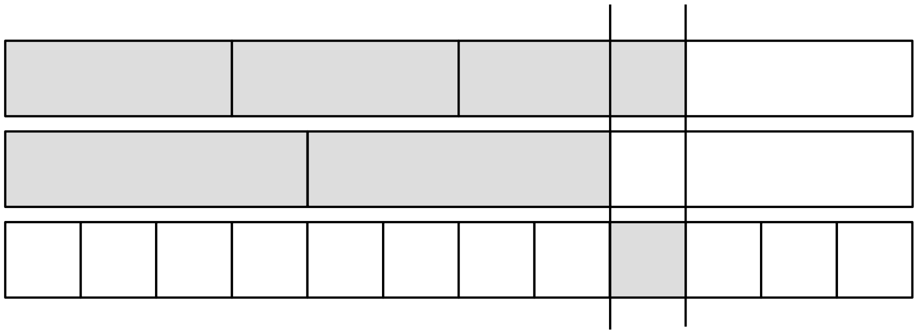
Question: work out $\frac{3}{4}-\frac{2}{3}$

Answer: choose a common denominator (12)

Write both fractions over the denominator: $\frac{3}{4}=\frac{9}{12}$ and $\frac{2}{3}=\frac{8}{12}$

Subtract the numerators: $\frac{9}{12}-\frac{8}{12}=\frac{1}{12}$

Visual presentation: you could use selected lines from a fraction wall to explore the equivalence and then visualise the difference…



Multiplying fractions

Multiplying fractions is easier than adding fractions!

Question: work out $\frac{3}{5}$ × 350

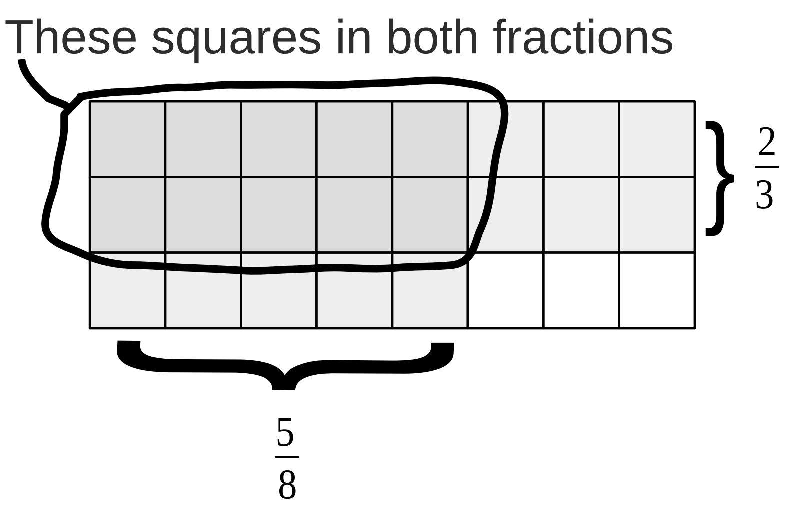
Answer: Multiplying a whole number by a fraction is the same as working out the value of the fraction so find $\frac{3}{5}$ of 350 in the usual way.

Question: work out $\frac{2}{3}\times\frac{5}{8}$

Answer: Multiply the tops, multiply the bottoms and simplify if needed

$$\frac{2}{3}\times\frac{5}{8}=\frac{2\times5}{3\times8}=\frac{10}{24}=\frac{5}{12}$$

Visual presentation: $\frac{2}{3}\times\frac{5}{8}$ means $\frac{2}{3}\text{of}\frac{5}{8}$ and the rectangle below attempts to show that $\frac{2}{3}\text{of}\frac{5}{8}$ must be $\frac{10}{24}$ …



Dividing fractions

Recall that dividing by a number smaller than 1 gives an answer that is *larger* than the number you started with, so 3 ÷ 0.2 = 15. Try it on your calculator. You need fifteen 20p coins to make £3.

Question: work out $\frac{3}{4} \div \frac{2}{15}$

Answer: <u>Keep</u> the first fraction, so $\frac{3}{4}$

<u>Flip</u> the multiplication into a division so $\frac{3}{4} \times$

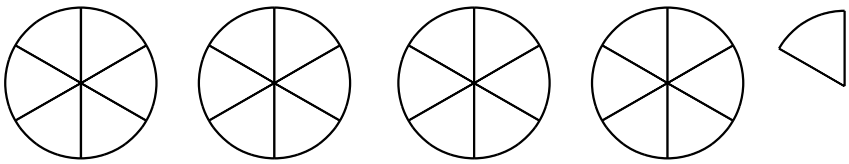
<u>Change</u> the fraction you are dividing by so it is the other way up $\frac{3}{4} \times \frac{15}{2}$

$$\frac{3}{4} \times \frac{15}{2} = \frac{45}{8} = 5\frac{5}{8}$$

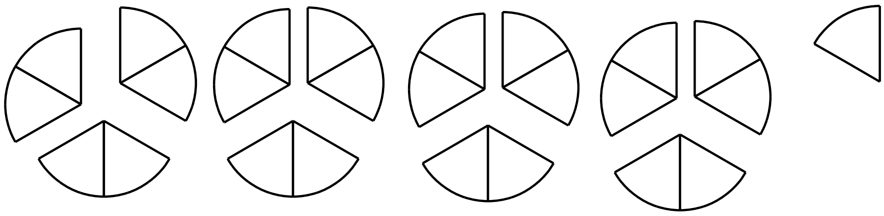
School children seem to remember the 'KFC' routine easily!

Visual presentation: work out $4\frac{1}{6} \div \frac{2}{3}$

Represent the $4\frac{1}{6}$ as pizzas...

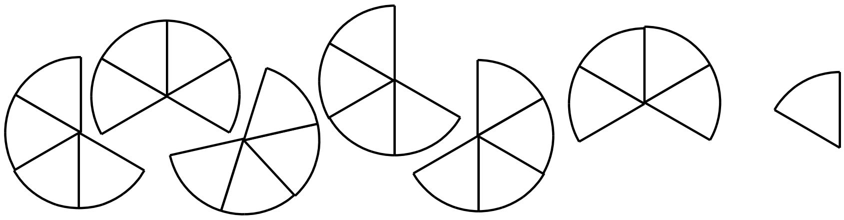


The calculation $4\frac{1}{6} \div \frac{2}{3}$ is asking you how many lots of $\frac{2}{3}$ are in $4\frac{1}{6}$ so group the pizza slices into thirds of a pizza.



So there are 12 lots of a third plus a slice left over. That single slice is half of a third (because a third is worth two slices).

Finally, group the thirds into groups of two...



So there are 6 lots of two thirds, plus that extra slice. That extra slice is one quarter of two thirds because each lot of two thirds has 4 slices. So the calculation works out to be $4\frac{1}{6} \div \frac{2}{3} = 6\frac{1}{4}$

NB: this method can work for tricker denominators but you end up having to sub-divide the slices.

Mixed Numbers and improper fractions

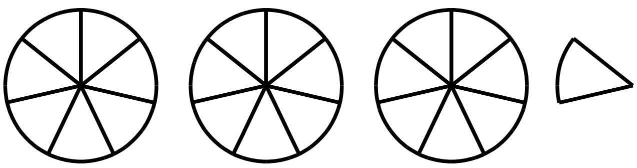
The number $3\frac{1}{7}$ is called a mixed number because there is a whole number followed by a fraction. $3\frac{1}{7}$ Means $3+\frac{1}{7}$. You can write this mixed number as an 'improper fraction' by converting the 3 wholes into 21 sevenths, and adding on the extra seventh: $3\frac{1}{7}=\frac{22}{7}$.

Convert a mixed number to an improper fraction

A **routine** for converting mixed numbers to top heavy fractions is

- Multiply the whole number by the bottom of the fraction
- Add the top
- Put the answer over the bottom.

A **visual presentation** could involve pizzas again…



Slicing the three whole pizzas into 7 slices each and adding the lone slice gives us 22 slices of size one seventh size, so $\frac{22}{7}$

Convert an improper fraction to a mixed number

A **routine** for converting improper fractions to mixed numbers is

- Divide the top by the bottom - you get a whole number and a remainder
- Write the whole number down
- Put the remainder over the bottom to get the fraction part

A **visual presentation** of converting $\frac{8}{3}$ to $2\frac{2}{3}$ would be counting up the slices to make whole pizzas…

Decimal arithmetic

Adding, subtracting, multiplying and dividing with decimal numbers is basically the same as with whole numbers. You need to keep track of the position of the decimal point when multiplying and dividing.

Adding and subtracting decimals

- Write the decimal numbers in columns with the decimal points underneath each other, and put a decimal point in the answer line.
- Fill in the spaces after the decimal point with zeros if it helps
- Add/subtract in columns as usual

Multiplying decimals

Question: work out 3.74 × 0.21

Answer: you ignore the decimal points, multiply as if whole numbers, then put the decimal point back in the right place afterwards...

- Count the total number of decimal places in both numbers (4 dp)
- Multiply both numbers without the decimal point (374 × 21 = 7854)
- Count back the total number of decimal places (Ans = 0.7854)

Why it works: When you treat 3.74 as 374, you have multiplied that number by 100. When you treat 0.21 as 21, you have multiplied that number by 100. So the answer will be 100 × 100 = 10 000 times too large so you have to divide by 10 000 to get the answer.

Dividing a decimal by a whole number

Question: Work out 37.5 ÷ 3

Answer: Just make a 'bus stop' with the 37.5 inside and the 3 outside as usual but put a decimal point on the top of the bus stop above the decimal point in the 37.5. Then just divide as usual working from the left and tracking the remainders.

Dividing by a decimal

Question: work out 18.72 ÷ 0.8

Answer: There isn't an algorithm for dividing by a decimal number so you multiply both numbers by enough powers of 10 so that the number you are dividing by becomes a whole number.

The steps in the routine are

- Count the number of decimal places in the number you are dividing by (1 dp)
- Move the decimal points to the left in both numbers by that number of decimal places so that you are dividing by a whole number (0.8 becomes 8 and 18.72 becomes 187.2)
- Divide as usual using a 'bus stop' (the calculation is 187.2 ÷ 8)
- There is no need to adjust the decimal place afterwards – we did the adjusting at the beginning

Why this works: Imagine a strange looking fraction $\frac{18.72}{0.8}$

Using equivalent fractions, we can make the bottom a whole number by multiplying both numerator and denominator by 10 so $\frac{18.72}{0.8}=\frac{187.2}{8}$ and now you can do the 'bus stop' division in the usual way. There is no need to adjust the decimal point afterwards because the fractions are equivalent.

Everyday arithmetic

In word problems there are key words and phrases that hint as to the operation to use.

Addition: sum, total.

Subtraction: difference, “how much more than”

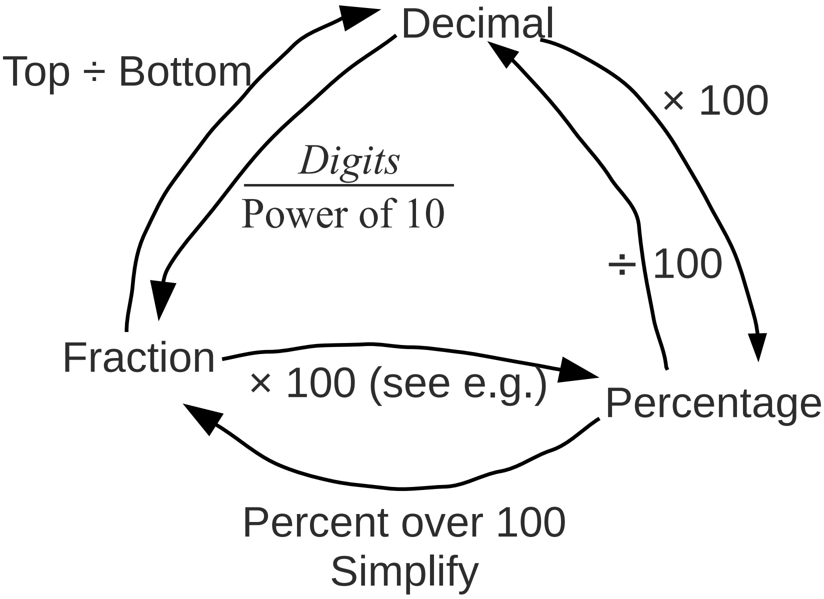
Multiplication: product, “£3 per unit how much is 25 units”, “Hourly rate is £7.80, how much for 12 hours?”

Division: quotient, “*how many* 250ml cups can you get from a 1.5 litre bottle of lemonade”

When you answer a word problem, always show the calculation you are doing in symbols, and then write the answer. Slips in arithmetic will cost you less marks that way.

Percentages, fractions and decimals

The diagram below summarises how you convert between decimals, fractions and percentages



The diagram summarises a lot of information. Examples of each of the conversions are shown below. You need to be confident with **all** of these for the exams, both with and without a calculator.

Fraction to decimal

Question: write $\frac{5}{8}$ as a decimal

Answer: Divide the top by the bottom. 5 ÷ 8 = 0.625

When the bottom is not a factor of 10, 100 or any power of 10, the result will be a recurring decimal, e.g. $\frac{4}{9}$ is 4 ÷ 9 = 0.444... or $0.\overline{4}$

Decimal to fraction

Question: write 0.65 as a fraction in its simplest form

Answer: 0.65 has two decimal places so write 65 over 100 and simplify

$$\frac{65}{100}=\frac{13}{20}$$

Decimal to Percentage

Question: Write 0.16 as a percentage

Answer: 0.16 × 100 = 16%

Just multiply by 100 to convert a decimal to a percentage.

Percentage to decimal

Question: Write 70% as a decimal

Answer: 70 ÷ 100 = 0.7

Divide by 100 to convert a percentage to a decimal fraction.

Fraction to percentage

Question: Convert $\frac{3}{5}$ to a percentage

Answer: Just multiply by 100 so $\frac{3}{5}\times\frac{100}{1}=\frac{300}{5}=60\%$

Another way of looking at this is to find $\frac{3}{5}$ of 100 which gives...

100 ÷ 5 = 20 × 3 = 60%

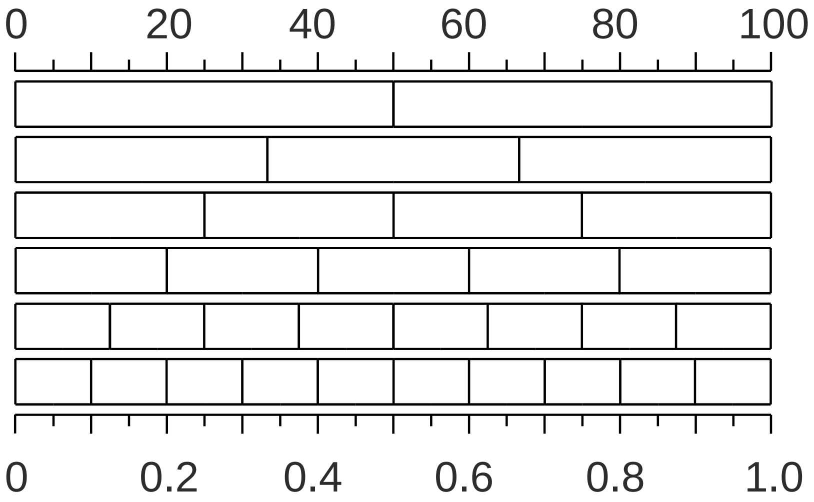
Percentage to fraction

Question: Write 35% as a fraction in its simplest form

Answer: Put 35 over 100 (that is what 'per cent' means after all) and simplify.

So 35% is $\frac{35}{100} = \frac{7}{20}$

Visual representation: the fraction wall below has a percentage scale and a decimal scale added so you can estimate the relative size of a given fraction, decimal or percentage.



Writing numbers as percentages

Question: Write 17 as a percentage of 20

Answer: Make a fraction $\frac{17}{20}$ then multiply by 100 so

$$\frac{17}{20} \times \frac{100}{1} = \frac{1700}{20} = 85\%$$

You need to read the questions carefully to decide which number to take as the denominator or bottom of the fraction. That number is the 'whole'.

Working out the percentage increase

Question: A first class stamp increases in price from 60p to 63p. Calculate the percentage increase.

Answer: The increase is 3p and 3 as a percentage of 60 is

$$\frac{3}{60} \times \frac{100}{1} = \frac{300}{60} = 5\,\%$$

You always write the increase as a percentage of the original amount, never the final amount.

Look for **increase words** like tax, VAT, surcharge, levy, profit and so on.

Working out the percentage decrease

Question: Narinder buys a car for £12 000 and then sells it for £9 000 a year later. What percentage of the value of the car has been lost?

Answer: Decrease is £3 000, and we want £3 000 as a percentage of £12 000. So $\frac{3000}{12000} \times \frac{100}{1}$. It is much easier if you simplify the fraction first, so $\frac{1}{4} \times \frac{100}{1} = 25\,\%$. Narinder lost 25% of the value of the car over the year he had the car.

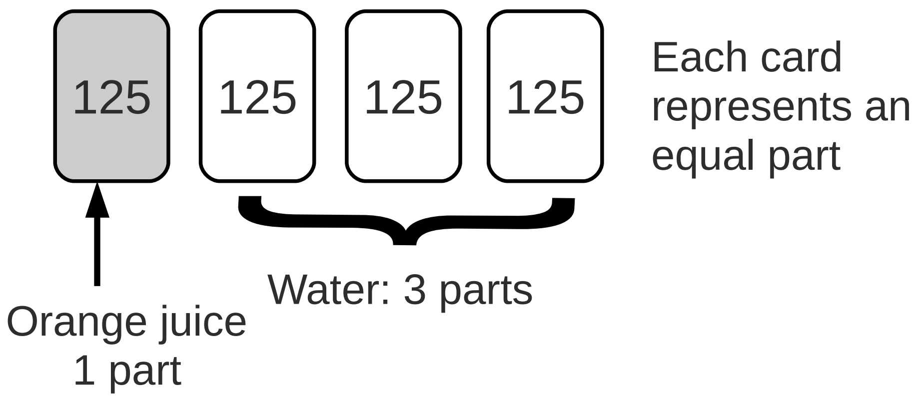
Look for **decrease words** like discount, reduction, loss, depreciation.

Ratio notation

Question: Ethel adds 125ml of orange juice to 375 ml of water to make a drink. Write down the ratio of orange juice to water in the drink in its simplest form

Answer: 125 : 375 is the ratio which simplifies to 1:3

Visual representation of the ratio can use the Singapore Bars or cards to represent each equal share…



Simplify ratios with fractions

Question: simplify the ratio $2\frac{1}{3}:1\frac{3}{5}$

Answer: This is a bit like comparing the fractions

- Make both mixed numbers top heavy $\frac{7}{3}:\frac{8}{5}$
- Write the fractions over the same denominator $\frac{7}{3}=\frac{35}{15}$ and $\frac{8}{5}=\frac{24}{15}$

  so the ratio becomes $\frac{35}{15}:\frac{24}{15}$
- Forget the denominator – the numerators give you the correct numbers to put in the ratio $35:24$

Simplify ratios with decimals

Question: simplify the ratio 4.5 : 7.25

Answer: Multiply all the ratio numbers by a power of 10 large enough to get rid of the decimal points, in this case 100

4.5 × 100 = 450 and 7.15 × 100 = 725

Make the new ratio: 450 : 725

Simplify the new ratio if needed: 450 : 725 simplifies to 18:29

Simplify ratios with units

Ratios are pure whole numbers - they don't have any units so you have to convert any units to the same unit before simplifying

Question: Simplify the ratio 3.5 m : 25 cm

Answer:

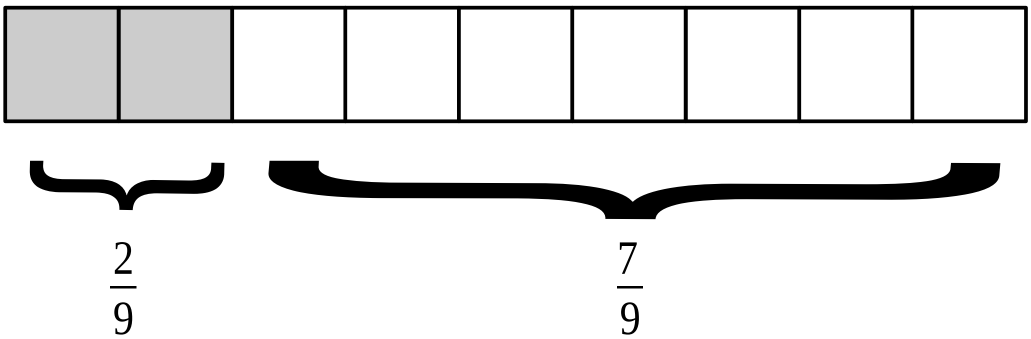
- <u>Convert both lengths</u> to the smaller unit so 350 cm: 25 cm
- <u>Forget the units</u> so the ratio is 350 : 25
- <u>Simplify as usual</u> so ratio becomes 14:1

Ratios and fractional parts

Question: A bag contains red and blue counters in the ratio 2:7
What fraction of the counters in the bag are red?

Answer: 2 + 7 = 9 so the denominator is 9. Two parts of the counters are red, so the numerator is 2. The fraction is $\frac{2}{9}$

Visual representation using a fraction bar…



Dividing in a ratio

Question: divide the number 360 in the ratio 5:7

Answer: Find out how many equal parts we know 5 + 7 = 12 equal parts.

Divide to find the value of an equal part: 360 ÷ 12 = 30

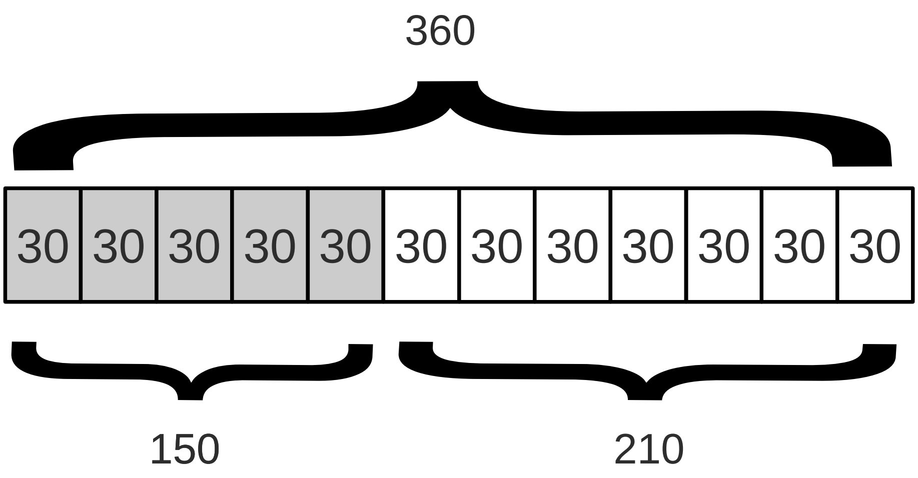
Multiply by each of the ratio numbers in turn to find the value

5 × 30 = 150 and 7 × 30 = 210

Check: add and compare to the original total 150 + 210 = 360. Correct!

Exam questions will have situations and story problems as well as simple direct questions. You need plenty of practice.

Visual representation and method: each card represents an equal part



Write what you know at the top

Decide how to find the equal value of each of the bars and label the bars

Count up the number of bars for one share and then the number of bars for another share. Put the totals at the bottom and check that they add up.

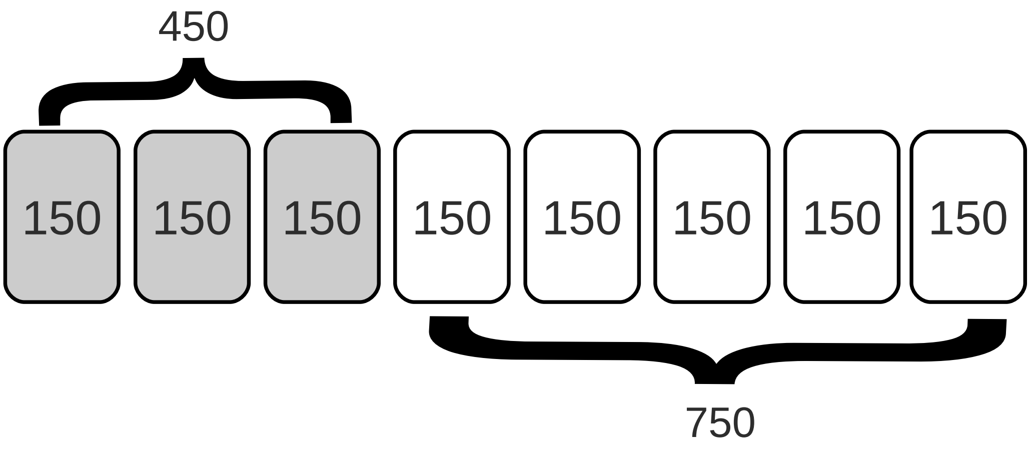
Ratio problems with one share known

Question: A drink is made by mixing juice and water in the ratio 3:5. Kamaljit has 450ml of juice. How much water does she need to add to make the drink?

Answer: Find out <u>how many equal parts we know</u>. 450 ml corresponds to 3 parts. You have to read the question carefully to find this information.

Find the <u>value of one equal part</u>: 450 ÷ 3 = 150 ml.
<u>Multiply</u> to find the value of the other part: 150 × 5 = 750 ml water

Visual representation and method: each card represents an equal part



- Work down the diagram
- Put the value of the share you know at the top grouped with the number of cards that make that share
- Label all the cards with the value of an equal share
- Count up the value of the other share

Ratio problems about differences

Question: Bill and Ben share the profits of a garden stall in the ratio 4:7.

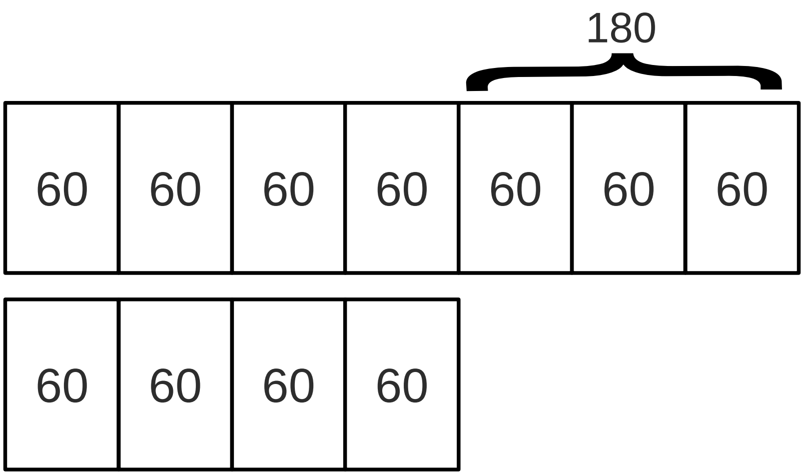
Ben receives £180 more than Bill one weekend. Work out the total profit that weekend.

Answer: Find the number of equal parts you know. 7 – 4 = 3 parts worth £180.

Find value of one equal part: £180 ÷ 3 = £60

Multiply to find the total: £60 × 11 = £660.

Visual presentation and method: this question uses a different layout for the bars to emphasise the difference between Bill and Ben's share…



- Work down the diagram
- Because the question is about the difference between the shares, I grouped the three more equal parts that Ben got with their value
- Work out what to write in each bar
- Count them all up to get the total

Write a ratio in the form 1:N

Ratios are usually made out of whole numbers, like 4:3 or 1:7 or even 2:5:9. Sometimes, you want a ratio in the form 1:N where N can be a whole number, a decimal or a fraction. This format is common in accountancy and in some aspects of baking as two examples of use.

Question: The ratio of profits to sales in a small company is 2:9. Write this ratio in the form 1: N.

Answer: We need to divide the other numbers in the ratio by the ratio number that corresponds to 1 in the format we want, so in this case we divide through by 2 and we get 1: 4.5 or 1: 4½

Proportion and Unitary Method

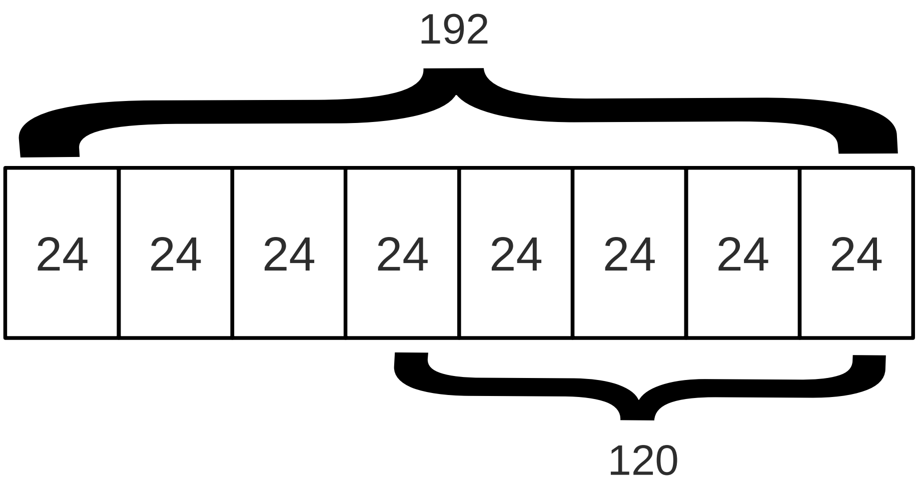
Question: 8 apples cost £1.92. Work out the cost of 5 apples

Answer: This is very like a ratio question

- Find cost of one apple by dividing 192 ÷ 8 = 24p each
- Multiply cost of one apple by the number you want 24 × 5 = £1.20

This method is called the Unitary Method because you find the cost of one apple first then multiply to find the cost of 5. The calculation is also very similar to finding the value of $\frac{5}{8}$ of the original.

Visual presentation and method: Bars help to see the logic of the Unitary Method



- Start at the top. We know that 8 apples cost 192p
- We can break 192 down into 8 equal parts
- Then you count up 5 of the equal parts
- The similarity to dividing up 192 in a ratio is clear

Recipe questions

Question: 450g of split peas needed for a recipe that serves 6. What weight of split peas do you need for 10 people?

Answer: Apply unitary method

- 450 ÷ 6 = 75g per serving
- 75 × 10 = 750g split peas needed

The visual method is the same as the Unitary Method illustration above.

Extra stuff: Tables

The multiplication tables are always useful, especially in week 4 when we cover multiplying decimals.

×	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

You need to have these numerical facts at your fingertips.

The work on factors and multiples in week 2 will provide practice.

Extra stuff: Metric units

A few of the questions on the exam will expect you to be able to convert between various metric units.

Below are some facts about metric units.

Length

- 10 millimetres (mm) in a centimetre (cm)
- 100 centimetres in a metre (m)
- So 1 000 millimetres in a metre
- 1 000 metres in a kilometre

Capacity (liquid measure)

- 1 000 millilitres (ml) in a litre (l)
- 1 000 litres in a cubic metre (m^3)

Weight

- 1 000 milligrams (mg) in a gram (g)
- 1 000 grams in a kilogram (kg)
- 1 000 kilograms in a tonne (T)